

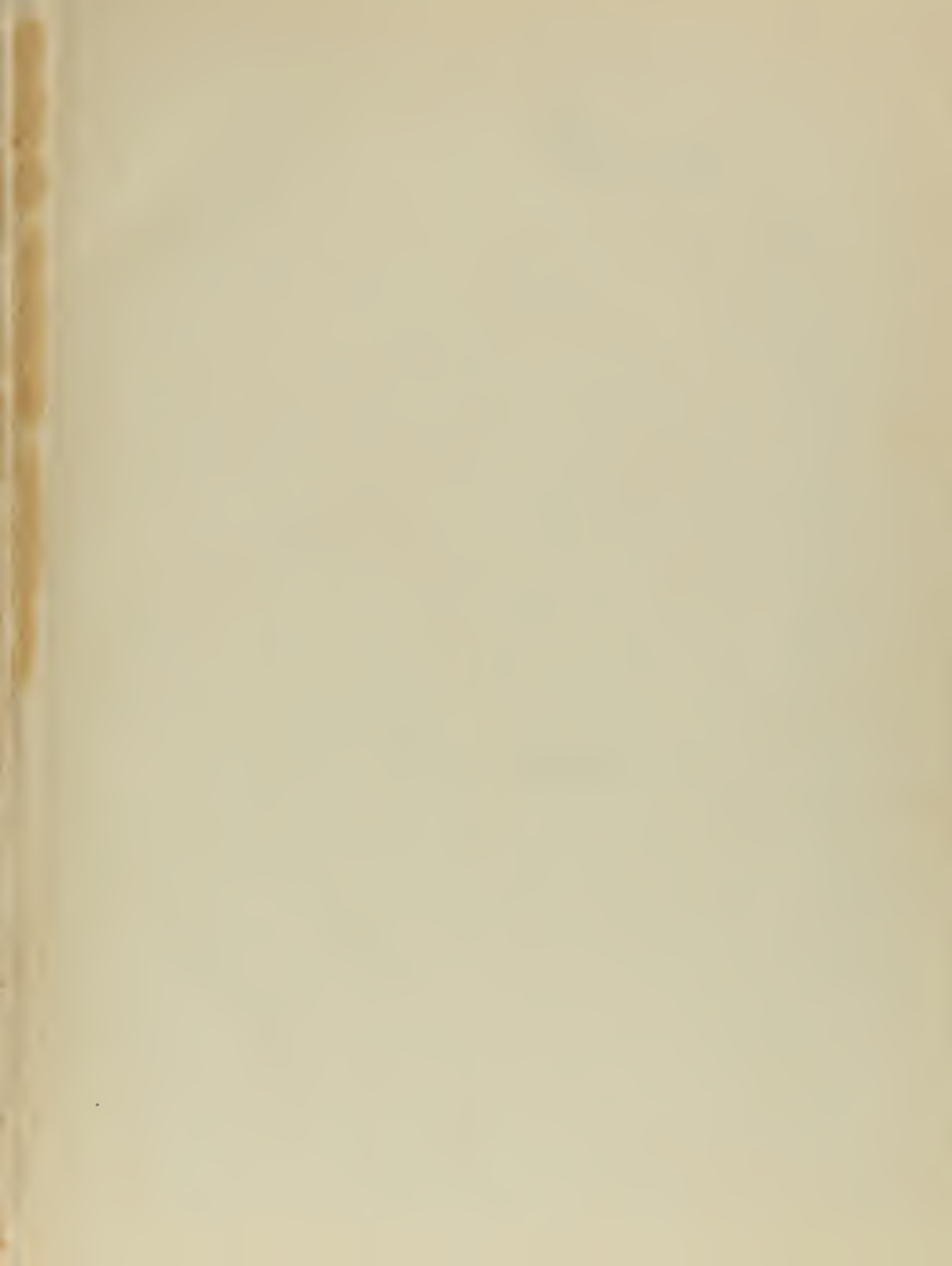
SINGLE SIDEBAND TRANSMISSION BY  
ENVELOPE ELIMINATION AND  
RESTORATION

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MICHAEL M. ELLIOTT

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SINGLE SIDEBAND TRANSMISSION  
BY  
ENVELOPE ELIMINATION AND RESTORATION

\* \* \* \* \*

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SINGLE SIDEBAND TRANSMISSION BY  
ENVELOPE ELIMINATION AND RESTORATION

by

Michael Murray Elliott  
Lieutenant, United States Navy

Submitted in partial fulfillment  
of the requirements  
for the degree of  
MASTER OF SCIENCE  
IN  
ENGINEERING ELECTRONICS

United States Naval Postgraduate School  
Monterey, California

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This work is accepted as fulfilling  
the thesis requirements for the degree of

MASTER OF SCIENCE  
IN  
ENGINEERING ELECTRONICS

from the  
United States Naval Postgraduate School



## PREFACE

December 12, 1901 was an eventful day in the history of radio communication. Early that afternoon in a bitter cold barracks on Signal Hill, St. Johns, Newfoundland, Guglielmo Marconi eagerly strained forward as he heard a faint sputtering in his headphones. The sputtering was caused by an interrupted spark gap electromagnetic radiator 1700 miles away in southwest England, and the intelligence conveyed by that sputtering was the letter "S" in International Morse Code. Ever since that day, scientists and engineers have been spurred on to develop more efficient, faster, and more flexible means of impressing intelligence on radio emissions. Many ingenious systems, some simple and some complex, have been devised to accomplish this modulation of the radio frequency emission.

It is the aim of this paper to show the evolution of one such system, single sideband transmission by envelope elimination and restoration. The writer sincerely believes that this system, invented by Leonard R. Kahn, a young engineering student, is one of the most promising developments in the field in recent years. It is believed that the system will find extensive application, not only in amateur and commercial use, but in the military sphere as well.

The writer wishes to thank Professor Earl G. Goddard of the U. S. Naval Postgraduate School for his assistance, encouragement and cooperation in the preparation of this paper.



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# TABLE OF SYMBOLS AND ABBREVIATIONS

(Listed in the order of their use in the text)

C.W.	- Continuous Wave
A.M.	- Amplitude Modulation or Modulated
D.S.B.	- Double Sideband
S.S.B.	- Single Sideband
R.M.S.	- Root Mean Square
A.G.C.	- Automatic Gain Control
R.F.	- Radio Frequency
S.S.S.C.	- Single Sideband, Suppressed Carrier
$\hat{E}_c$	- Peak Carrier Voltage Amplitude
$m_a$	- Modulation Factor
$\omega_c$	- Two pi times the carrier frequency
$\omega_m$	- Two pi times the modulation frequency
P.M.	- Phase Modulation or Modulated
D.C.	- Direct Current
A.F.C.	- Automatic Frequency Control
$e_g$	- Instantaneous Grid Voltage
$i_p$	- Instantaneous Plate Current
L - C	- Inductance - Capacitance
F.M.	- Frequency Modulation or Modulated
I.P.A.	- Intermediate Power Amplifier
R - C	- Resistance - Capacitance
P.A.	- Power Amplifier
k	- A constant
$\hat{E}_m$	- Peak Modulating Voltage Amplitude
$J_n$	- Bessel Function, first kind, $n^{\text{th}}$ order



## CHAPTER I

### INTRODUCTION

Electromagnetic radiation, as postulated by James Clerk Maxwell and experimentally verified by Hertz and Marconi, has grown from a physical oddity to a multi-million dollar business and an indispensable aid in the world-wide transmission of intelligence. The basic building block upon which all radio communication rests is the modulation of the electromagnetic radiation; that is, the means for impressing intelligence on the transmission. Given the radio transmitter, the operator can energize it causing it to emit electromagnetic waves which may be received at a distant point. The recipient, however, has gained no intelligence from this reception, other than the fact that there is a radiation present in the spectrum. The transmitting operator must vary the radiation in accordance with some pattern which conveys the desired intelligence to the recipient. This variation is called the modulation of the radio wave.

All of the methods of modulation of a continuous radio frequency wave are based on the principle of varying some characteristic of that wave in accordance with the intelligence to be transmitted. The characteristics of an R.F. wave are: its presence in the spectrum, its amplitude, its frequency, and its phase. Variation of the first constitutes the familiar on - off keying known as C.W. Variation of any of the latter three constitute the basic methods for the transmission of audio intelligence. This paper will be primarily concerned with an important modification to the





basic theory of amplitude modulation, in the interests of bettering its efficiency as a means of conveying intelligence. In the process of discussing these modifications, it will be seen that basic phase modulation theory will also enter into the discussion; therefore Appendix I, analytically demonstrating the principles of amplitude and phase modulation, has been included.

This important means for bettering the efficiency of A.M. is the use of single sideband suppressed carrier transmission. This paper will discuss the principles of such transmission; its evolution from A.M.; basic means for its generation; and its principal advantages and disadvantages. The paper will then take up in detail a system for the generation of single sideband waves using a conventional A.M. transmitter as the medium for building up the power of such a wave. The theory behind this system (single sideband by envelope elimination and restoration) will be given and practical design considerations will be discussed. A proposed application of the system to a U. S. Navy A.M. transmitter will be described, and experimental results and conclusions will be set forth.





## CHAPTER II

### SINGLE SIDEBAND SUPPRESSED CARRIER TRANSMISSION

In amplitude modulation, as has been shown in Appendix I, all of the intelligence is contained in either sideband; the carrier adds nothing to the intelligence; and two thirds of the total power in the transmitted amplitude modulated wave is contained in the carrier. Examination of the above statements leads one logically to ask, "Why not eliminate both the carrier and one of the two sidebands?" Intelligence-wise, this elimination will not change the characteristics of the wave at all, and since we are interested only in conveying intelligence from point to point, the scheme immediately looks feasible. Several substantial advantages in using such a system immediately become apparent, and their existence and degree of benefit will be shown here in detail.

This, then, is single sideband — the transmission of an amplitude modulated wave with the carrier reduced or removed and one sideband eliminated. Naturally, there are many practical problems involved in the generation and reception of such a wave; but the superiority of the system over ordinary amplitude modulation will be obvious after a review of its advantages.

The first improvement over ordinary A.M. is self-evident from the definition of single sideband. If one sideband is removed from an amplitude modulated wave, its bandwidth is reduced by a factor of one half. Thus,



for a given bandwidth, use of single sideband allows the transmission of twice as much intelligence, say through the use of multiplex. It would also allow transmission of higher fidelity signals if desired.

If the problem of reception is examined for a moment, another advantage over amplitude (D.S.B.) modulation appears. The signal to noise ratio of an ideal receiver is defined as<sup>(1)</sup>:

$$\frac{E_s^2}{4kT_aRB} \quad (\text{a power ratio})$$

Where  $E_s$  is the available signal voltage at the receiver input terminals,  $k$  is Boltzmann's Constant,  $T_a$  is the effective absolute temperature of the radiation resistance of the antenna,  $R$  is the equivalent noise resistance of the antenna, and  $B$  is the bandwidth. The higher this ratio for a given signal, the greater the ability of the receiver to receive weak signals. Hence, if one halves the bandwidth by using S.S.B., the receiver signal to noise ratio can be doubled, giving a 3 db increase power-wise in receiver effectiveness.

The greatest benefit realized through the use of S.S.B., however, is derived from the fact that the carrier has been radically reduced or eliminated. After eliminating the carrier and one sideband, the total transmitter output power is available for conveying intelligence. To illustrate quantitatively the benefits derived from this situation, a comparison will be drawn<sup>(2)</sup> between an amplitude modulated transmitter and a single sideband suppressed carrier transmitter of the same peak amplitude capacity. By the latter is meant the maximum signal amplitude which the

(1)  $\frac{1}{\sqrt{1-\beta^2}}$

$$\frac{1}{\sqrt{1-\beta^2}}$$



transmitter can produce with an acceptable amount of distortion. For the A.M. transmitter at 100% modulation the maximum peak amplitude handled will occur when the carrier, upper sideband, and lower sideband all add in phase. Letting the carrier have a peak amplitude of unity, the sidebands then each have a peak amplitude of one half. Adding the three in phase produces a sum of two, which is now assumed as the peak amplitude capacity of the given transmitter. The R.M.S. value of this amplitude is 1.414. The R.M.S. amplitude of the intelligence-bearing portion of the signal, however, is 0.707 ( $0.5 + 0.5$ ), or 0.707. With the single sideband signal, assuming the same transmitter peak amplitude capacity of two, all of the intelligence is contained within the one signal transmitted. Hence, the R.M.S. amplitude of the intelligence-bearing "portion" of the S.S.B. signal is  $0.707(2)$ , or 1.414. Therefore, the ratio of R.M.S. amplitudes of the intelligence carried by S.S.B. to that of A.M. is two to one. Since power is proportional to the square of the R.M.S. voltage, this gives a six db power advantage to the S.S.B. transmitter. With six db of effective signal to noise power increase due to the elimination of the carrier and one sideband (placing all of the intelligence in the other sideband) and a three db increase in signal to noise ratio due to halving the required receiver bandwidth, single sideband suppressed carrier communication provides a theoretical nine db improvement in signal to noise ratio over that obtained with double sideband with carrier (A.M.) transmission.



Selective fading does not have the deleterious effect on a single sideband transmission that it does on D.S.B. transmission. In A.M. double sideband transmission selective fading of the carrier results in the receiver demodulating the intelligence with respect to the dominant sideband component, generating distorting cross product terms and also producing apparent overmodulation. In S.S.B. transmission the carrier is supplied at the receiver and hence cannot fade; the fading of one component of a complex voice sideband will not be noticeable. Selective fading of the carrier in A.M. will also cause loss of A.G.C. and resultant blasting; this will not occur in S.S.B. reception.

Since there is no appreciable carrier in a single sideband wave, the annoying type of interference between A.M. stations spaced closely together, known as heterodyning, will not be a problem.

During intervals when there is no intelligence impressed on the carrier in A.M. transmission (assuming broadcast rather than break-in operation) 67% of the transmitter power is still being radiated, and hence wasted as far as the recipient is concerned. When a single sideband transmitter is not being modulated, the only power still being radiated is that contained in the suppressed carrier, which is negligible. Hence, large savings in power may be effected through the use of S.S.B.

As will be seen in the discussion of methods of generating the S.S.B. wave, all of the modulation is done at low power levels, and hence savings in modulator equipment costs are substantial.

1877-1878. The first of the following years was 1877.

The second year was 1878. The third year was 1879.

The fourth year was 1880. The fifth year was 1881.

The sixth year was 1882. The seventh year was 1883.

The eighth year was 1884. The ninth year was 1885.

The tenth year was 1886. The eleventh year was 1887.

The twelfth year was 1888. The thirteenth year was 1889.

The fourteenth year was 1890. The fifteenth year was 1891.

The sixteenth year was 1892. The seventeenth year was 1893.

The eighteenth year was 1894. The nineteenth year was 1895.

The twentieth year was 1896. The twenty-first year was 1897.

The twenty-second year was 1898. The twenty-third year was 1899.

The twenty-fourth year was 1900. The twenty-fifth year was 1901.

The twenty-sixth year was 1902. The twenty-seventh year was 1903.

The twenty-eighth year was 1904. The twenty-ninth year was 1905.

The thirtieth year was 1906. The thirty-first year was 1907.

The thirty-second year was 1908. The thirty-third year was 1909.

The thirty-fourth year was 1910. The thirty-fifth year was 1911.

The thirty-sixth year was 1912. The thirty-seventh year was 1913.

The thirty-eighth year was 1914. The thirty-ninth year was 1915.

The fortieth year was 1916. The forty-first year was 1917.

The forty-second year was 1918. The forty-third year was 1919.

The forty-fourth year was 1920. The forty-fifth year was 1921.



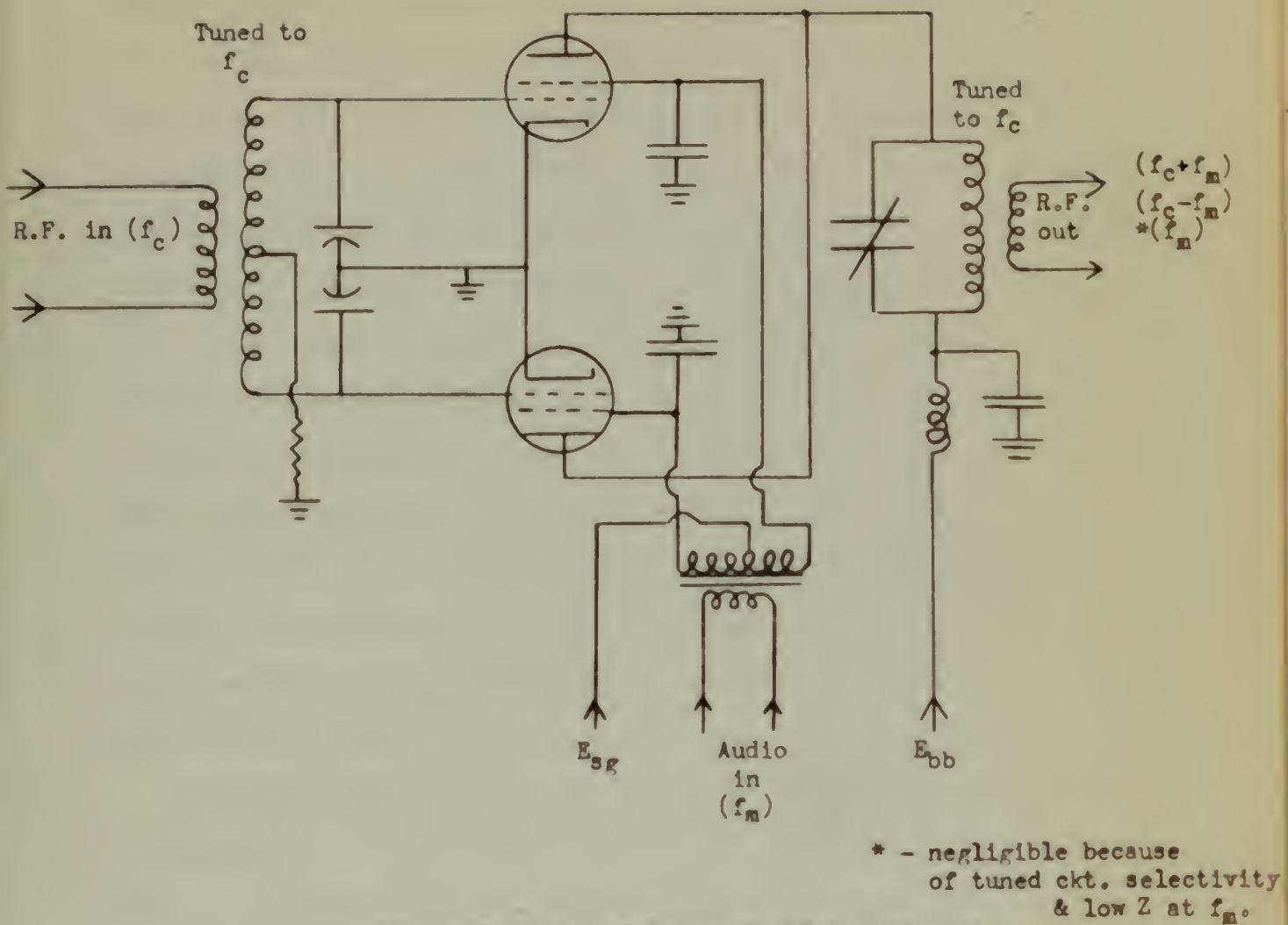
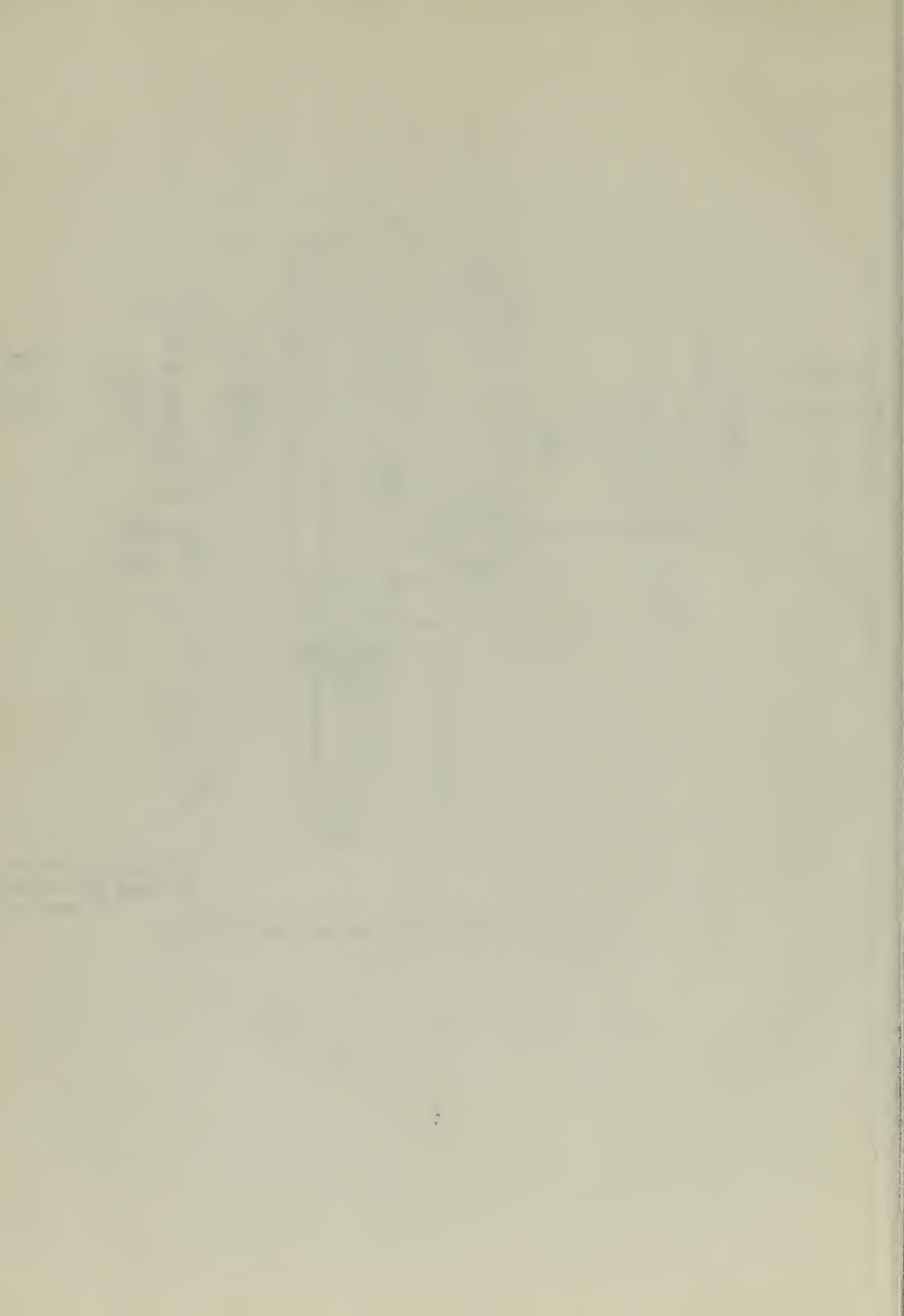


Figure 1 - Basic Balanced Modulator Circuit



Until the advent of the systems to be described in Chapter III, there were two basic methods for generation of single sideband suppressed carrier emissions. Both methods utilize the balanced modulator for elimination or suppression of the carrier component. The balanced modulator is simply an amplifier in which the intelligence and the carrier are combined as excitations in such a manner that the carrier will be cancelled in the output, leaving only the upper and lower sidebands. It may be aptly compared<sup>(3)</sup> with the old "push-push" type of doubler amplifier, in which the R.F. carrier input is applied to the grids in push-pull and the plates are connected in parallel to the output tank circuit (tuned to the fundamental frequency in this case). If the audio intelligence is applied to the tubes (say to the screens in the case of tetrodes) in push-pull, then with zero audio input, the carrier components cancel in the plate load and there is no output. When audio is applied, it upsets the balance at the audio rate, causing the sidebands only to appear in the output. One basic circuit for such a balanced modulator is shown in Figure 1.

The ring modulator, essentially a diode bridge circuit, is also used to eliminate the carrier component in the generation of single sideband waves. Its basic circuit, together with associated frequencies, is shown in Figure 2. The potentiometer has been found necessary to achieve good balance; the rectifying elements are usually copper oxide varistors. The ring modulator is usually used at low frequencies<sup>(4)</sup>.



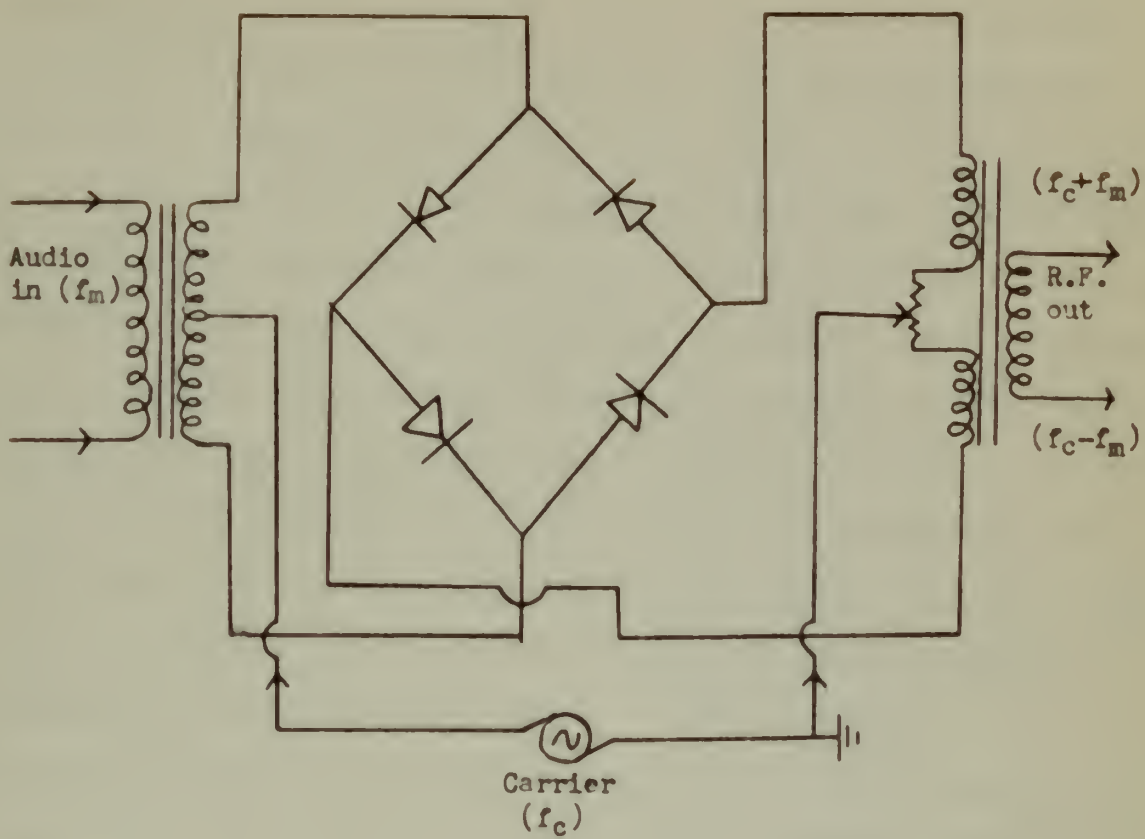
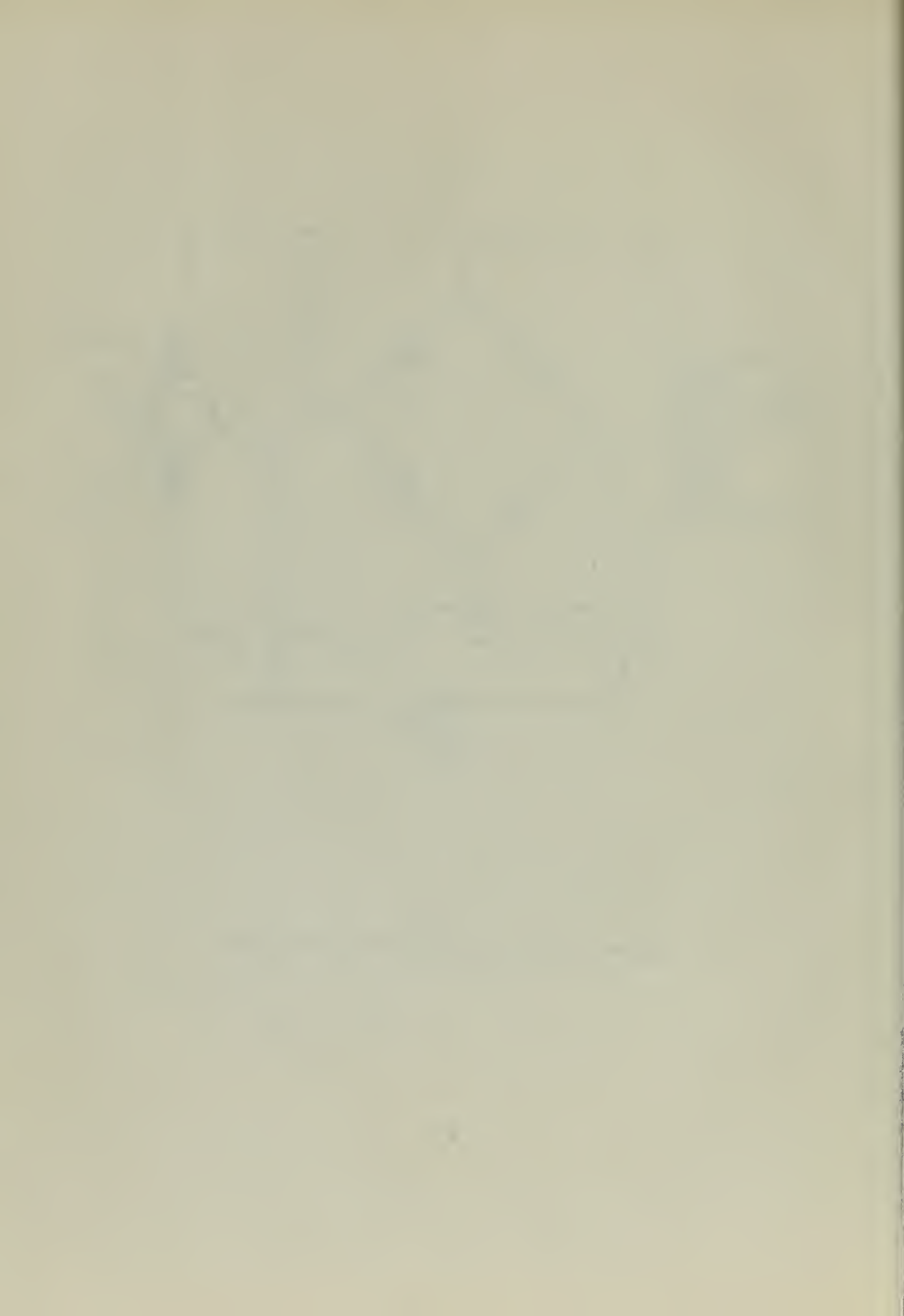


Figure 2 - Basic Circuit of Ring Modulator





Although the method used for eliminating the carrier is common to both systems, the difference between the two shows up in the means for eliminating the undesired sideband. The first solution to the problem of eliminating a relatively narrow band of frequencies would naturally be the use of a bandpass filter acting on the desired sideband. This then is the filter or "brute force" method of generating S.S.B.; the intelligence is introduced and the carrier suppressed in a balanced modulator, and one of the two resulting sidebands is suppressed through the use of a bandpass filter. Inasmuch as practical design limitations complicate the construction of sharp cutoff filters above the ultrasonic or low radio frequencies, several balanced modulators and bandpass filters may be employed in cascade to heterodyne the original signal up to the desired output radio frequency. A complete block diagram of a filter type S.S.S.C. generator together with associated sample spectra is shown in Figure 3. It is important to note that all functions indicated in the block diagram may be accomplished at a low power level, giving definite economic advantages.

The other basic method for generation of S.S.S.C. emissions is called the phasing method. It also utilizes balanced modulators for suppressing the carrier, but the unwanted sideband is phased out rather than filtered out. The practical approach to this method is best shown by simple mathematics.

Given an amplitude modulated wave expressed as

$$e_1 = \hat{E}_c \sin \omega_c t + \frac{\hat{E}_c m_o}{2} \cos (\omega_c - \omega_m) t - \frac{\hat{E}_c m_o}{2} \cos (\omega_c + \omega_m) t = \hat{E}_c [1 + m_o \sin \omega_m t] \sin \omega_c t$$

$$\frac{d}{dt} \left( \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \right) = \frac{d}{dt} \left( \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \right)$$



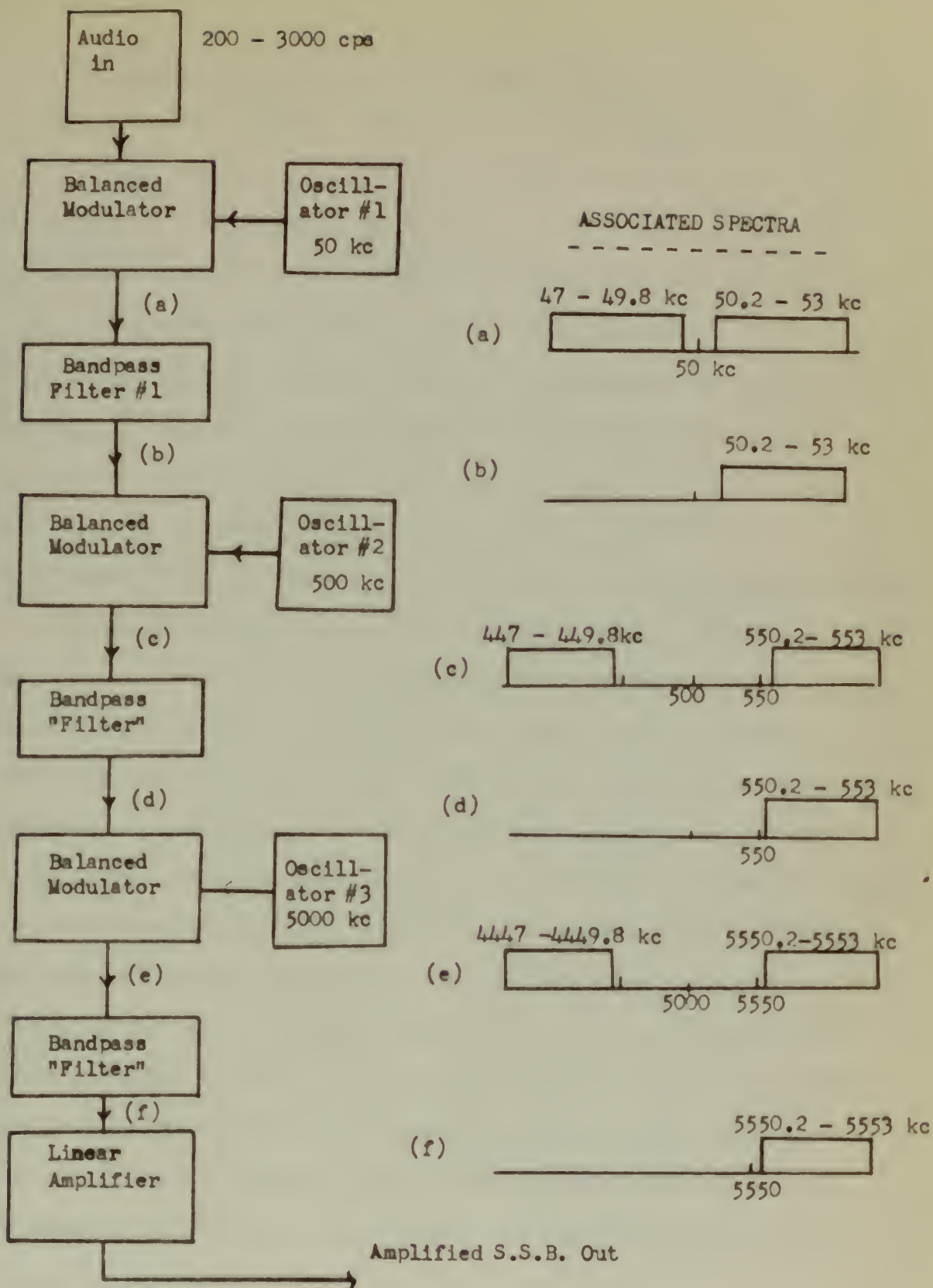


Figure 3 - S.S.B. Generation; Filter Method



and a second amplitude modulated wave expressed as,

$$e_2 = \hat{E}_c \cos \omega_c t + \frac{\hat{E}_c m_a}{2} \cos (\omega_c - \omega_m) t + \frac{\hat{E}_c m_a}{2} \cos (\omega_c + \omega_m) t = \hat{E}_c [1 + m_a \cos \omega_m t] \cos \omega_c t$$

simply add the two together, obtaining

$$e_1 + e_2 = \hat{E}_c [\sin \omega_c t + \cos \omega_c t] + \hat{E}_c m_a \cos (\omega_c - \omega_m) t = \sqrt{2} \hat{E}_c \sin [\omega_c t + 45^\circ] + \hat{E}_c m_a \cos (\omega_c - \omega_m) t$$

which is a single sideband wave with the lower sideband and carrier present. If they had been subtracted, the result would have been

$$e_1 - e_2 = \hat{E}_c [\sin \omega_c t - \cos \omega_c t] - \hat{E}_c m_a \cos (\omega_c + \omega_m) t = \sqrt{2} \hat{E}_c \sin [\omega_c t - 45^\circ] - \hat{E}_c m_a \cos (\omega_c + \omega_m) t$$

which again is single sideband with the carrier and upper sideband present.

Looking at the right side of the equations for  $e_1$  and  $e_2$  it is seen that in order to achieve this elimination of the sideband, two amplitude modulated waves must be supplied in which the carrier and modulation of one is  $90^\circ$  out of phase with those of the other. After adding or subtracting, the remaining carrier component can be suppressed as in the filter system with a balanced modulator. In practice, the audio and the carrier are applied directly to one balanced modulator, giving the upper and lower sidebands as one output. The same audio and carrier are also shifted  $90^\circ$  in phase and then combined in a second balanced modulator giving an output in which one of the two sidebands is  $180^\circ$  out of phase with that coming from the first balanced modulator. These two outputs are added, giving a resultant single sideband output with no carrier, assuming perfect balance. This is shown in the phasing system block diagram of Figure 4.

$$2\left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

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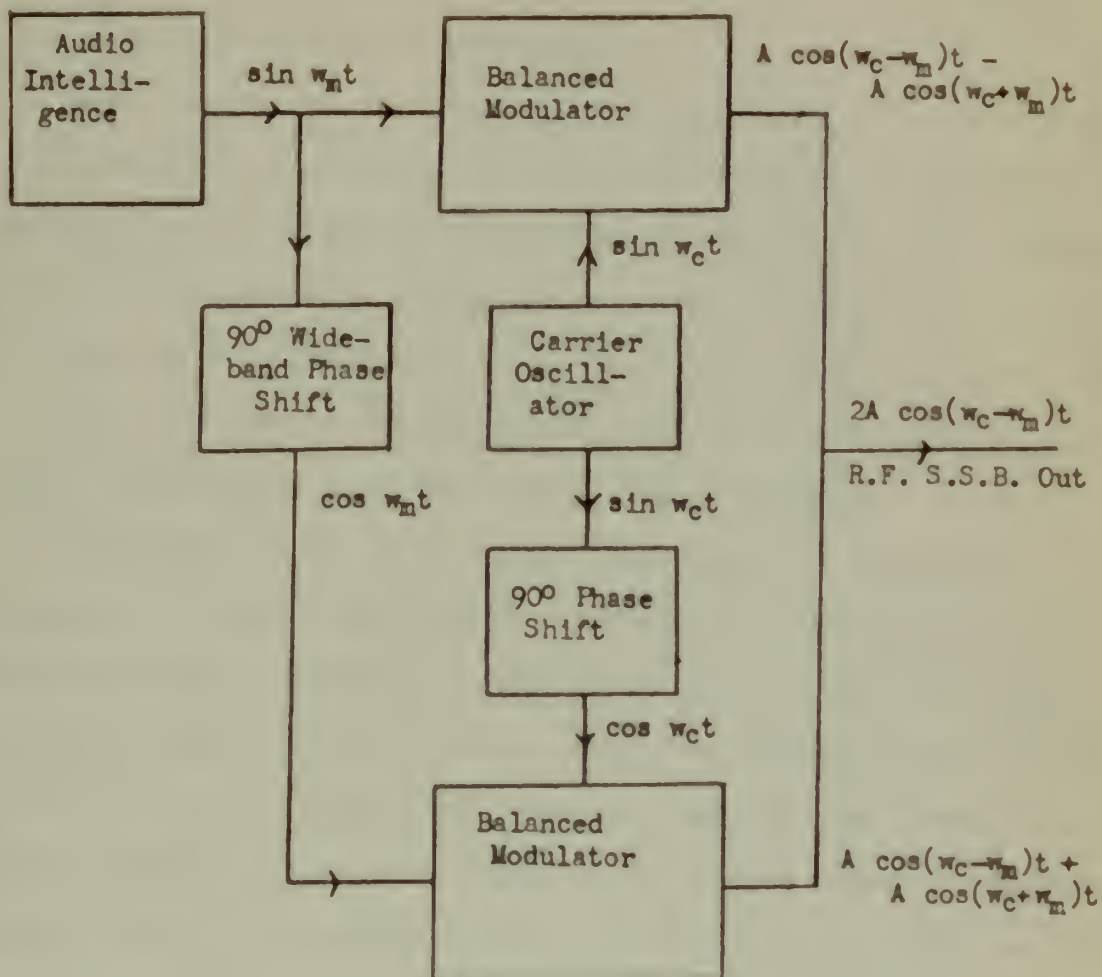
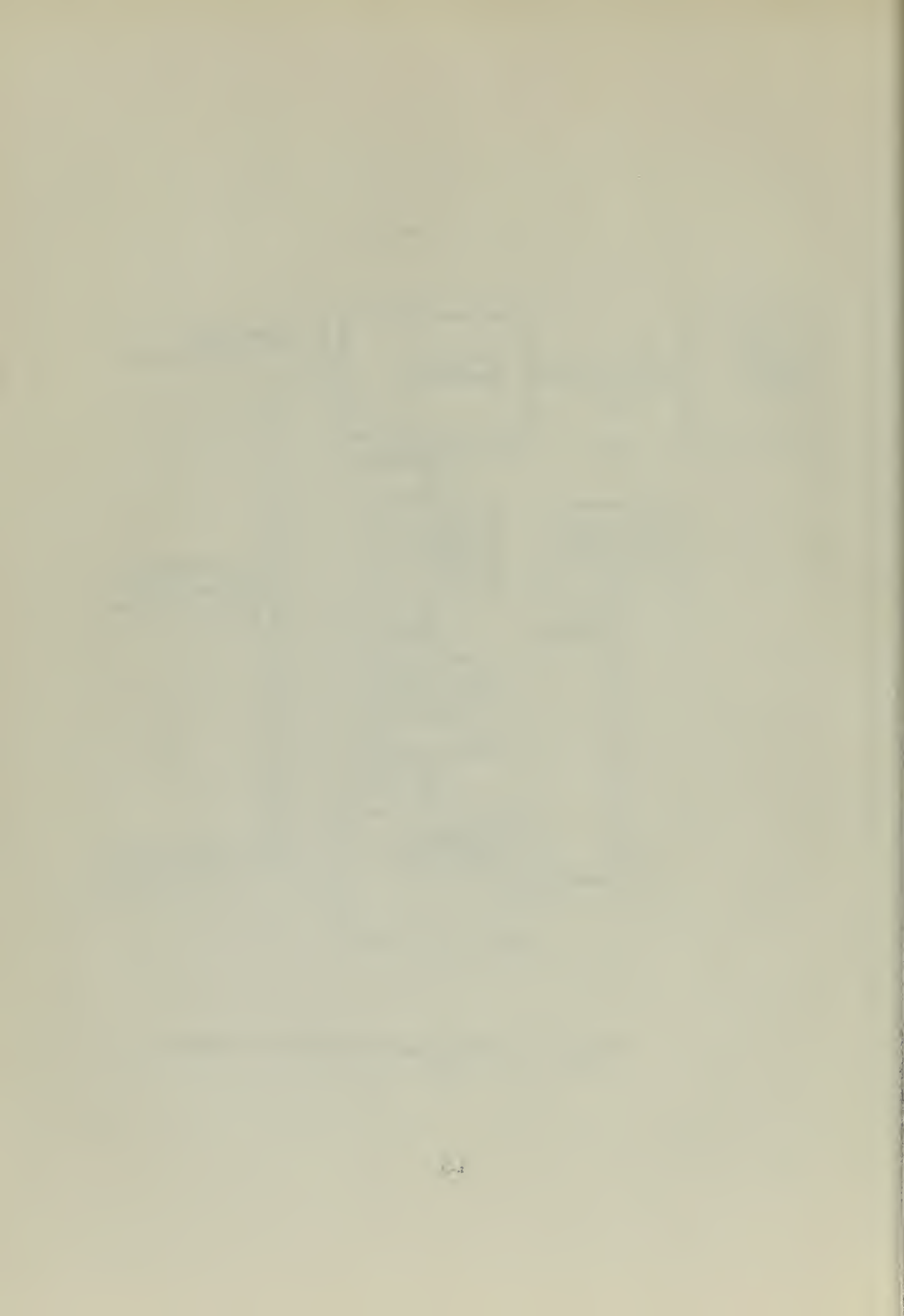


Figure 4 - Phasing Method for S.S.B. Generation





The  $90^\circ$  phase shift of the carrier is simple to achieve, since the carrier frequency does not vary appreciably; but shifting a wide band of audio frequencies uniformly across the band is more difficult. It can be done, however, and ingenious means for doing so, such as the Dome wideband phase shift network<sup>(5)</sup> have been devised.

The filter system of S.S.S.C. generation is simple and straightforward; it has the distinct advantage that once the filtering suppression has been achieved, the use of passive elements in the filters precludes subsequent need for adjustment over a long period of time. It has the disadvantage that with the present state of the art of filter design, sideband elimination must take place at low frequencies, and hence more stages must be employed to heterodyne up to the desired output frequency.

The phasing system seems a great deal simpler at first glance. The mathematics, however, show that the amplitudes of the two waves  $e_1$  and  $e_2$  must be the same; this implies no insertion loss or gain in the phase shift networks, which is hard to realize. Also, the audio phase shift must be  $90^\circ$  over the entire audio spectrum; this has already been mentioned as extremely hard to achieve. In order to arrive at the correct output amplitudes for cancellation of the unwanted sideband, the balanced modulators employed must be identical; this is impossible in practice, but careful design will approach this condition. As might be inferred, adjustment of this type of system is critical, and it must be checked frequently for adequate sideband and carrier suppression.

Single sideband suppressed carrier transmission has several disadvantages, of course. In order to achieve appreciable power after generation



of the S.S.B. wave, linear amplifiers must be employed to preserve the waveform of the intelligence. This limits the overall efficiency of the system and dictates more exacting operating conditions and adjustments. If these linear amplifiers are cascaded (as they may be for high power), the individual amplifier must be extremely linear to avoid cascading the distortion of the wave. Since the received S.S.B. signal must be demodulated with respect to the carrier frequency, frequency instability of receiver carrier resupply circuits is a major problem. It is usually met by transmitting a small amount of carrier along with the sideband and using this to synchronize a resupply circuit in the receiver. Reception of S.S.B. signals on conventional A.M. receivers is impossible unless such receivers have a stable means of carrier resupply, such as a beat frequency oscillator.

The foregoing covers the basic principles of single sideband transmission; points out the advantages to be gained in using it; briefly describes basic methods for the generation of such a wave; and summarizes the disadvantages of conventional single sideband transmission. Considering the undisputed fact that single sideband transmission is being used more and more extensively for high reliability on communication circuits, one must infer that the advantages outweigh the disadvantages. Moreover, any single sideband system that has all the advantages of the conventional ones described and at the same time eliminates one or more of the disadvantages would be well worth investigating. The remainder of this paper will be devoted to such an investigation.





### CHAPTER III

#### SINGLE SIDEBAND BY ENVELOPE ELIMINATION AND RESTORATION—EVOLUTION AND BASIC CONCEPTS

It was pointed out in the previous chapter how advantageous the use of single sideband transmission can be over conventional A.M., especially with regard to the apparent signal to noise power gain over comparably rated A.M. equipment. In order to realize these advantages and at the same time generate appreciable power, there seems to be one big disadvantage — the cascading of linear amplifiers with their attendant critical operating conditions and possible multiplication of individual amplifier distortions. Also, the present A.M. user who wishes to convert his equipment so that he can enjoy the increased benefits of S.S.B. would have to sacrifice a large part of his investment in that A.M. equipment. This is apparent from the basic design differences between the methods of generating the two types of waves. If some system could be devised that generated high power single sideband emission, did not require the use of linear amplifiers, and could easily be adapted to existing A.M. equipment, all of the advantages of S.S.B. could be retained and the worst disadvantages nullified.

Before going into systems with the above characteristics, it is necessary to examine the basic structure of a single sideband wave. The results gained from such examination will point toward the method of attack in meeting the above requirements.

# THE HISTORY

OF THE UNITED STATES OF AMERICA

FROM 1776 TO 1876

BY JOHN P. HARRIS, LL.D.,

OF THE UNIVERSITY OF CHICAGO, ILLINOIS.  
NEW YORK: PUBLISHED BY THE  
AUTHOR, 15 N. 5TH ST., N.Y.C.  
1876.  
THE HISTORY OF THE UNITED STATES OF AMERICA, FROM 1776 TO 1876, BY JOHN P. HARRIS, LL.D., OF THE UNIVERSITY OF CHICAGO, ILLINOIS. NEW YORK: PUBLISHED BY THE AUTHOR, 15 N. 5TH ST., N.Y.C. 1876.

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A single sideband wave produced by single tone modulation of the carrier may be written as

$$e = A_1 \cos \omega_c t + A_2 \cos (\omega_c + \omega_m) t$$

where  $A_1$  is the peak carrier amplitude and  $A_2$  is the upper sideband peak amplitude.

Expanding this expression yields (6)

$$\begin{aligned} e &= A_1 \cos \omega_c t + A_2 [\cos \omega_c t \cos \omega_m t - \sin \omega_c t \sin \omega_m t] \\ &= [A_1 + A_2 \cos \omega_m t] \cos \omega_c t - [A_2 \sin \omega_m t] \sin \omega_c t \\ &= \sqrt{A_1^2 + 2A_1 A_2 \cos \omega_m t + A_2^2 \cos^2 \omega_m t + A_2^2 \sin^2 \omega_m t} \cos \{ \omega_c t + \varphi \} \end{aligned}$$

$$\text{Where } \varphi = \arctan \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t}$$

$$= \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \omega_m t} \cos \{ \omega_c t + \varphi \}$$

Thus, the addition of the carrier and one sideband results in a cosinusoidal wave whose amplitude and phase both vary with the modulation.

The amplitude of the S.S.B. wave is  $\sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \omega_m t}$

The phase angle of the S.S.B. wave is  $\arctan \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t}$

Let  $\mathcal{A}$  be a collection of sets, and let  $\mathcal{B}$  be a collection of sets.

Let  $\mathcal{C}$  be a collection of sets.

$$\mathcal{A} \cup \mathcal{B} = \{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$$

Let  $\mathcal{D}$  be a collection of sets, and let  $\mathcal{E}$  be a collection of sets.

Let  $\mathcal{F}$  be a collection of sets.

Let  $\mathcal{G}$  be a collection of sets.

$$\{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\} = \{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$$

$$\{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\} = \{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$$

$$\{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\} = \{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$$

$$\frac{A \cup B}{A \cap B} = \frac{A \cup B}{A \cap B}$$

$$\{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\} = \{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$$

Let  $\mathcal{H}$  be a collection of sets, and let  $\mathcal{I}$  be a collection of sets.

$$\{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\} = \{A \cup B \mid A \in \mathcal{A}, B \in \mathcal{B}\}$$

$$\frac{A \cup B}{A \cap B} = \frac{A \cup B}{A \cap B}$$

From this it can be seen that a single sideband emission is both amplitude and phase modulated by the intelligence. Such a wave would have an instantaneous frequency given in Appendix II. The result shows that the S.S.B. wave frequency is a function both of the modulation amplitude and the modulating frequency, just as it is for a phase modulated wave as demonstrated in Appendix I.

The foregoing analysis immediately points the way toward amplification of single sideband without the use of linear amplifiers, and will confirm an approach toward utilization of standard A.M. transmitters in the amplification of such a wave. It is a well known fact that class C amplifiers will amplify a purely phase modulated wave without distorting the intelligence; and since the envelope of the single sideband wave is varying at an audio rate, it would seem that the envelope of the wave could be amplified by conventional speech amplifiers working into standard A.M. modulators. The problem is to correlate these facts into an acceptable system.

In 1948 Villard<sup>(7)</sup>, having recognized the advantages of single sideband and also the apparently insurmountable difficulties of converting an A.M. transmitter into the conventional form of single sideband transmitter, developed a system which would generate a S.S.B. type signal with carrier. His system was designed to apply the principle to existing A.M. or P.M. transmitters.

The system utilized what has been proved in the foregoing analysis; that a single sideband signal with carrier is composed of both amplitude and phase modulation components. Essentially, the Villard system was this: a C.W. carrier was generated, it was then phase modulated, and finally the

There is a great deal of work to be done in the field of

the study of the history of the United States, and it is

very important that we should have a good knowledge of

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resulting wave was amplitude modulated in the class C final. A block diagram of the system appears in Figure 5. The part enclosed in dashed lines comprises the conventional A.M. transmitter; the remaining blocks are all that are necessary to accomplish the desired result. The reason for the phase shift and frequency correcting networks will become clear in the following explanation.

The carrier wave is generated by the usual crystal oscillator, V1. It drives a conventional class C buffer amplifier stage V2, and at this point the carrier is phase modulated by the audio. The phase modulated output of V2 then drives the final amplifier, which in turn is amplitude modulated by the audio signal. The resultant output is then a single sideband wave with carrier.

In order to achieve cancellation of the unwanted sideband, the phase modulation must have the correct phase relationship to the amplitude modulation<sup>(8)</sup>. Figure 6(a) shows the usual spectrum of amplitude modulation at the instant the modulation cycle is passing through zero. Figure 6(b) shows the spectrum of a narrow band phase modulated wave produced by the same intelligence that produced 6(a). Note that this is an accurate portrayal of phase relationships, since A.M. sidebands and first order P.M. sidebands produced by an identical modulating signal are inherently in phase quadrature. This has been demonstrated in Appendix I. The additional 90° audio phase shift necessary to bring about the desired spectrum shown in Figure 6(c) is accomplished in the Dome wideband phase shifting network shown. Addition of the spectra of Figures 6(a) and 6(c)





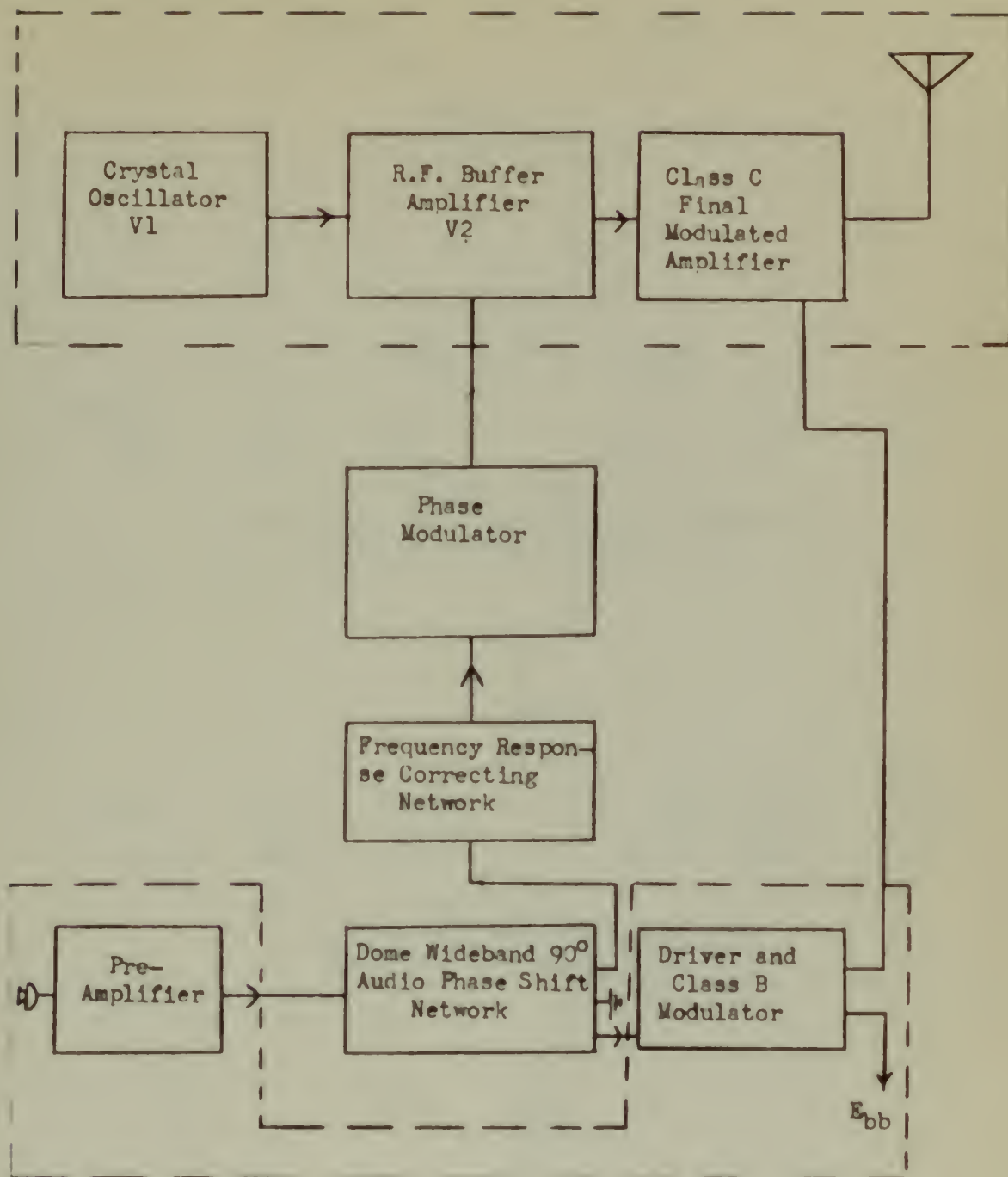


Figure 5 - S.S.B. With Carrier Generation; Villard System



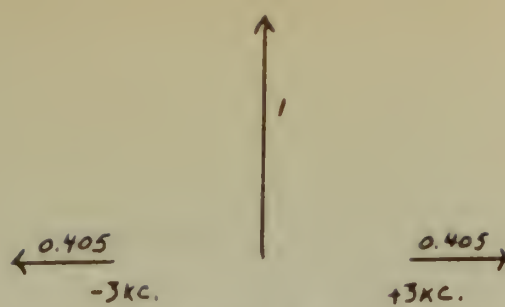


Figure 6 (a)  
Conventional A.M. Spectrum

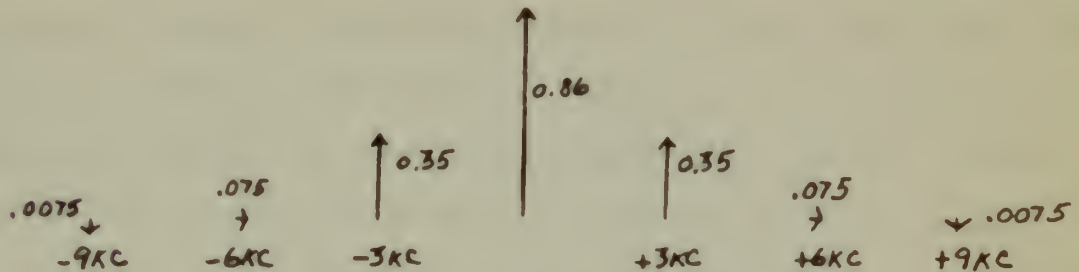


Figure 6 (b)  
Conventional P.M. Spectrum - Same Modulating  
Voltage as Figure 6 (a)

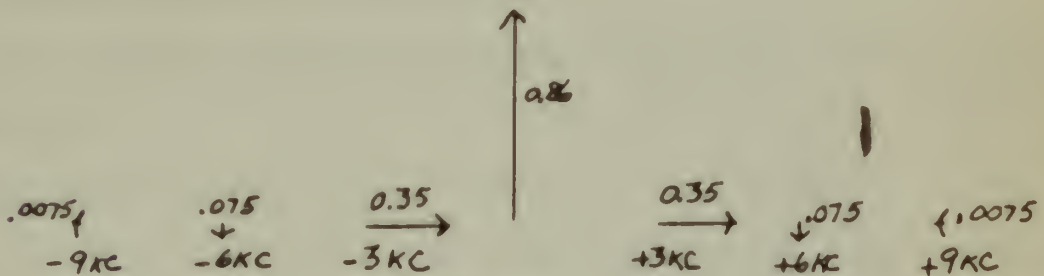


Figure 6 (c)  
P.M. Spectrum Produced by 90° Audio  
Phase Shift in Figure 6 (b)

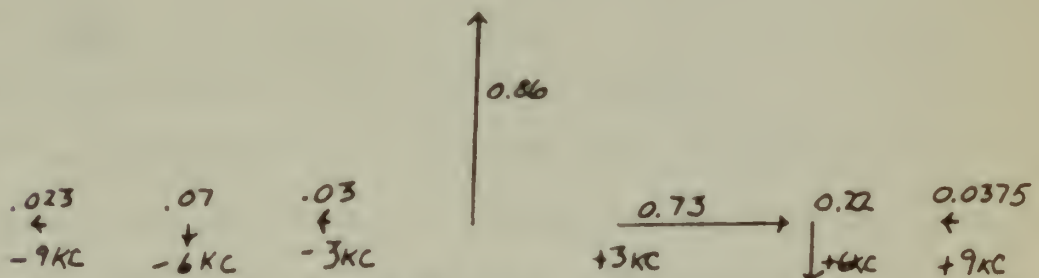


Figure 6 (d)  
Resultant Spectrum When Adding 6 (c) to 6 (a)

Figure 6 - Derivation of Spectrum for Villard System



results in the final spectrum shown in Figure 6(d), which is a single sideband type wave with carrier. Villard specifies the indicated amplitudes based on an A.M. signal at 81% modulation and a phase modulation of 0.75 radians. With these operating conditions it can be seen that he has achieved a sideband suppression of about 24 to one, or 27.7 db. Note that the A.M. modulation power has been reduced to  $(0.405)^2/(0.5)^2$ , or 65.9% of that required for 100% straight amplitude modulation. The R-C frequency compensating network shown is necessary to make the frequency response of the phase modulator equal to that of the amplitude modulator. It also helps to limit the overall bandwidth at the higher modulating frequencies.

The advantages of this system are:

- (a) Bandwidth occupied (disregarding higher order sidebands) has been halved.
- (b) Less modulator power is required.
- (c) Can be added to any A.M. transmitter with only minor modifications.
- (d) Avoids the use of balanced modulators and their attendant problems.
- (e) Can be received on any conventional receiver, since the carrier is still present. A.G.C. can be employed.

The disadvantages of this system are as follows:

- (a) The resultant output is not true S.S.B., and hence not all the advantages of such emission can be realized. For instance, the carrier has not been suppressed; heterodyning with nearby signals will still be





present, and power economy has not been fully realized. Villard, of course, knew this and specified the design merely as an interim between A.M. and S.S.B.

(b) Use is made of a wideband phase shift network which requires precision components and careful design and construction.

(c) Sidebands of higher order than the first are not suppressed on the "accepted" side of the carrier. This would give rise to adjacent channel interference and also a broader bandwidth than that of true S.S.B. The relatively large second order sideband on the accepted side does serve a useful purpose in that it removes the usual distortion present in the audio output of a diode detector when receiving a carrier and one sideband; this is true only when the second order sideband is within the receiver passband. When outside (at high modulating frequencies), it does not help prevent such distortion and will cause adjacent channel interference.

The logical transition, then, is to a system which generates, amplifies, and emits true single sideband suppressed carrier, still adaptable to conventional A.M. transmitters but eliminating the disadvantages of the system just described. Kahn<sup>(9)</sup>, investigating the problem in 1952, arrived at a solution to it, proposing the alternate approach that led to single sideband transmission by envelope elimination and restoration. As with Villard's system, the Kahn approach depends on the composite phase and amplitude modulation present in a single sideband suppressed carrier wave.

The approach in brief is this:

(1) A single sideband suppressed carrier wave is generated conven-



tionally by either of the two methods described in Chapter II of this paper.

(2) The S.S.B. output wave from this generator is then limited, removing all the amplitude variations and leaving only the phase modulation which is inherently present in the wave.

(3) The resulting phase modulated wave of constant amplitude is then amplified by conventional class C amplifiers in the A.M. transmitter and applied to drive the final modulated class C amplifier.

(4) Meanwhile, the output of the S.S.S.C. generator is also fed to a detector, which produces an audio output corresponding to the envelope or amplitude modulation component of the S.S.B. wave.

(5) This audio reproduction of the S.S.B. wave envelope is amplified in the regular speech amplifier and driver of the A.M. transmitter and is then applied to drive the A.M. modulator.

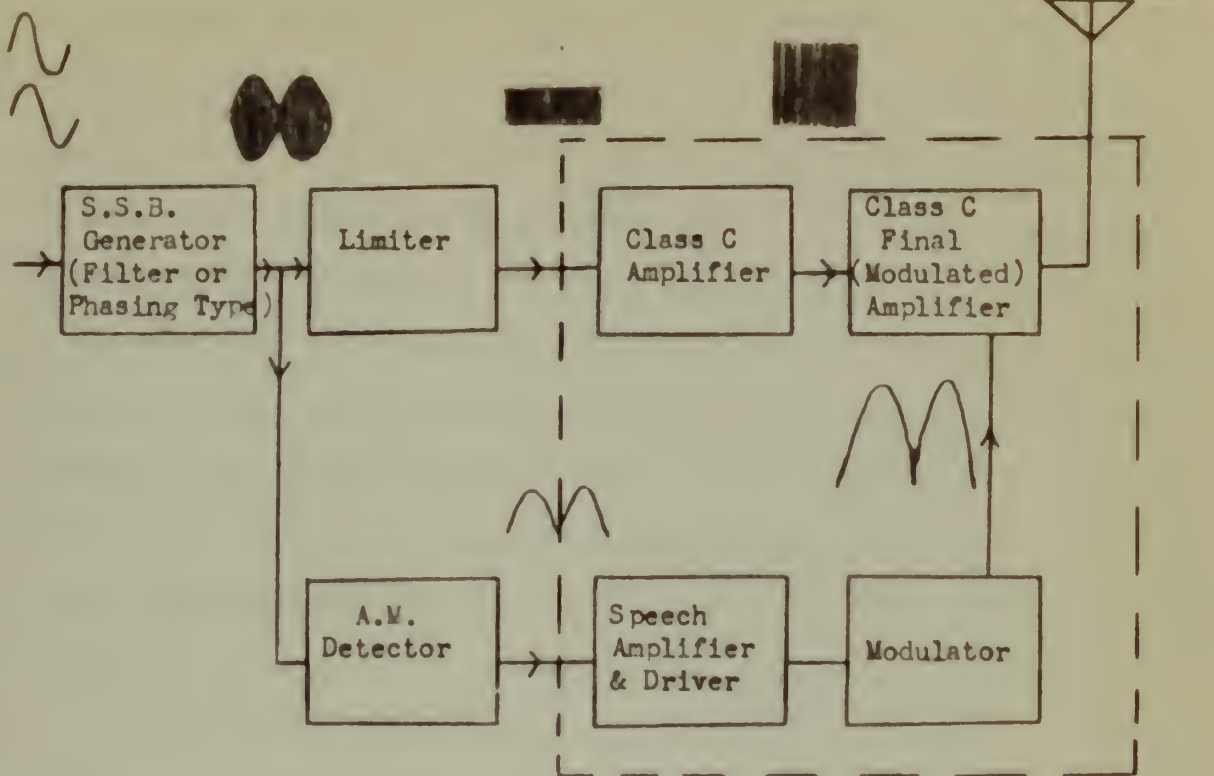
(6) The final class C amplifier output (the amplified phase modulation component of the S.S.B. signal) is then amplitude modulated by the output of the A.M. modulator, superimposing the amplitude variation on the phase variation and hence restoring the envelope of the S.S.B. wave.

(7) The resultant output at the antenna is then a faithful reproduction of the original generated S.S.S.C. wave, greatly amplified.

The simplified block diagram, showing the sequence of the above steps, appears as Figure 7. Waveforms shown depict the result of modulating the S.S.B. generator with two equal amplitude tones of different audio frequency, the usual standard test input to a S.S.B. generator.

(1) The first of these is the fact that the  
 (2) second is the fact that the  
 (3) third is the fact that the  
 (4) fourth is the fact that the  
 (5) fifth is the fact that the  
 (6) sixth is the fact that the  
 (7) seventh is the fact that the  
 (8) eighth is the fact that the  
 (9) ninth is the fact that the  
 (10) tenth is the fact that the  
 (11) eleventh is the fact that the  
 (12) twelfth is the fact that the  
 (13) thirteenth is the fact that the  
 (14) fourteenth is the fact that the  
 (15) fifteenth is the fact that the  
 (16) sixteenth is the fact that the  
 (17) seventeenth is the fact that the  
 (18) eighteenth is the fact that the  
 (19) nineteenth is the fact that the  
 (20) twentieth is the fact that the





Notes: (a) Shaded waveforms indicate phase modulated R.F.

(b) Portion enclosed by dashed lines is conventional A.M. transmitter.

Figure 7 - S.S.B. by Envelope Elimination and Restoration;  
Basic Block Diagram





The immediate advantages of the system are apparent. First, since this is true single sideband suppressed carrier transmission, all of the advantages of such emission listed in Chapter II are realized with the possible exception of savings in modulator equipment costs. The problem of what happens during standby periods when no intelligence is being transmitted will be taken up later. Secondly, this is a system which may be adapted to conventional A.M. transmitters, as was Villard's. Third, there are no low efficiency linear amplifiers with which to contend. Hence, distortion and spurious frequency generation are independent of transmitted power level. Fourth, the complete system external to the A.M. transmitter may be designed compactly with receiving type tubes used throughout, making it ideal as an accessory or an adapter to a large A.M. transmitter.

A more detailed examination of the system is now necessary. It is evident that if the output at the antenna is to be a faithful reproduction (amplified) of the output of the S.S.B. generator, then the phase relationship between the envelope of the wave and the phase modulated portion must be preserved. In other words the time delay of the wave travel through the P.M. branch must equal the time delay of wave travel (the detected envelope) through the A.M. branch.

The question then arises as to the time delay introduced by each branch. If each stage of the P.M. branch works into a pure resistance (a low  $Q$  load at resonance), there will be essentially no time delay in that stage; that is, when the grid voltage goes up, the plate load voltage will go down simultaneously. With a high selectivity plate load circuit, however,



a reactive component will be present along with the resistive component for frequencies slightly off resonance, and the load voltage would lead or lag the exciting grid voltage. This would introduce a time delay in the branch. For practical purposes assuming the typical low Q load for class C stages that is purposely designed into A.M. transmitters, the time delay for the P.M. branch may be neglected.

The modulator or A.M. branch, however, will have largely reactive loads and hence plate load voltage per stage will lead or lag the grid voltage for that stage in time. This condition, together with the fact that the modulator operates at a much lower frequency (or a longer time period for one modulation cycle) will result in appreciable time delay between exciting and output voltage. Therefore, the time delay of the A.M. branch (readily measurable for a given transmitter) must be compensated for by adding a time delay circuit in the P.M. branch.

At radio frequency, time delay is easily accomplished through the use of lumped circuit parameter delay lines. The amount of time delay required is easily determined by comparing the modulator input signal with its output signal (single tone modulation) and measuring the time difference between corresponding points on the two waves. The delay line would then be designed for that time difference.

The problem of what happens when the average D.C. amplitude of the generated single sideband wave varies will now be considered. With frequency shift (S.S.B.) keying or steady tone modulation, the average





amplitude of the wave would be constant, but with voice modulation, the average D.C. amplitude of the generated single sideband wave would vary.

Looking at the block diagram in Figure 7, it is seen that the limiter removes all amplitude variations from the generated S.S.B. wave; hence a variation in average D.C. amplitude of that wave would not be transmitted to the P.M. branch. Naturally it should not, since the P.M. branch consists of class C amplifiers which require constant excitation. The envelope or audio variation would be applied by the A.M. branch; but it would be superimposed on a phase modulated signal of constant average amplitude, rather than on one which is varying in average amplitude. Therefore, the output wave would not be a copy of the generated wave. To correct this some means must be incorporated for varying the amplitude of the output S.S.B. wave in accordance with variations in the average D.C. amplitude of the generated S.S.B. complex voice wave. The output of the detector may be amplified, rectified, and the audio variations filtered out, providing a D.C. control voltage that is proportional to the average D.C. amplitude of the S.S.B. wave. This voltage would be used to control (through appropriate circuitry) the voltage at one of the electrodes of the final class C amplifier so that the amplitude of the output wave at the antenna would follow the variations in the average amplitude of the generated wave. In effect, one can consider the final class C amplifier as modulated by two agencies: (1) the A.M. modulator, which superimposes the envelope or instantaneous amplitude on the carrier, and (2) the output level control, which superimposes the average amplitude variations on the carrier.





There is also a completely different approach to the problem of average amplitude variation. In this approach, the amount of amplitude modulation would determine the amount of carrier suppression; that is, as the amplitude of the modulating voltage fell, some external agency would make the carrier amplitude rise, with the circuitry so adjusted that the average of the S.S.B. wave generated would remain constant. This would result in a large carrier being transmitted during periods of silence which would allow the improved use of squelch, AGC, and AFC at the receiving end. However, it is believed that such an alternate system would defeat several basic purposes and advantages of S.S.S.C. transmission, such as suppression of heterodyne interference, economy of power, and containment of intelligence in all appreciable radiation. This seems to be too big a price to pay for achieving the advantages listed, and will not be considered here further.

Naturally, the necessary mixers and oscillators may be added to P.M. branch to heterodyne the S.S.B. signal up to the desired operating frequency. Here, however, one notes an important limitation. In the class C stages of the A.M. transmitter there must be no multiplying; they must be operated straight through. If frequency multiplication is used, the phase deviation is also multiplied, making the output wave an inexact copy of the input wave.

All of the above necessary additions to the theoretical block diagram are shown in their proper placement in Figure 8. Frequencies listed are taken for convenience only to illustrate the relative ranges used in the design.



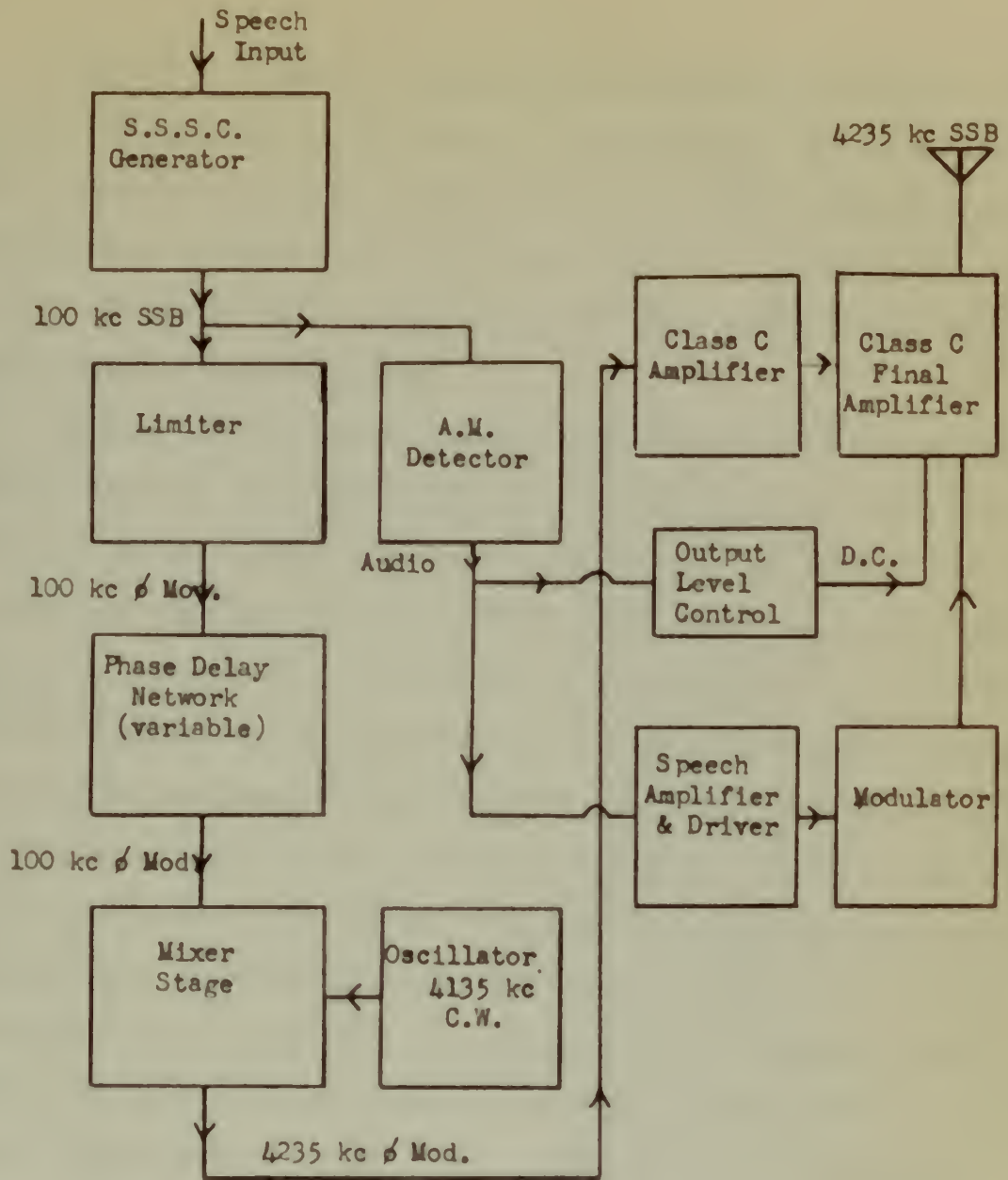
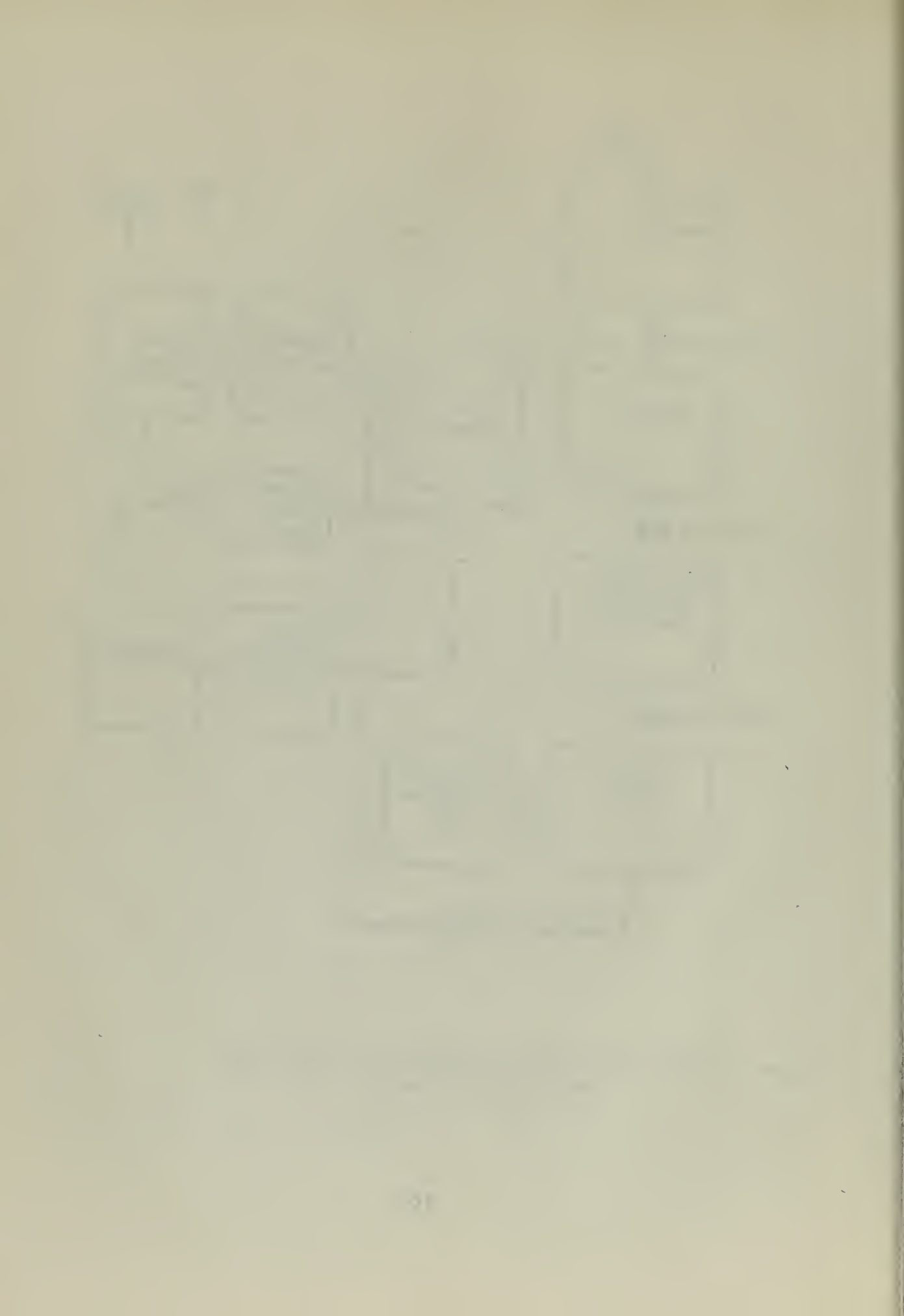


Figure 8 - S.S.B. by Envelope Elimination and Restoration;  
Practical Block Diagram





Inasmuch as there are several methods of amplitude modulation that can be designed into conventional A.M. transmitters, the question naturally arises as to whether single sideband by envelope elimination and restoration is compatible with all these methods. One might infer from the block diagrams that the system must be built around an A.M. transmitter that is plate modulated.

Representative practical types of modulation usually encountered (besides plate) are: control grid, screen grid, suppressor grid, and cathode modulation. All of these forms operate on the principle of varying the voltage impressed on one or more of the modulated amplifier's electrodes at an audio rate. They depend on the amplifier's tube characteristics to translate this variation into a proportional variation of the R.F. carrier amplitude. The several forms of grid modulation are characterized by a reduction in the carrier power output obtainable from a given R.F. amplifier tube, as well as by rigorous operating adjustments and requirements. These disadvantages are partially offset by the decided advantage that very low power is required from the modulator, allowing compact, light-weight, and inexpensive design. In grid modulation, the plate voltage remains constant, and application of the modulation in series with the grid electrode voltage varies the plate current and efficiency of the modulated amplifier. Thus, assuming linearity of the tube characteristic ( $e_g - i_p$ ), the carrier voltage developed across the resonant plate load will vary in amplitude exactly as the modulating voltage does.<sup>(10)</sup>



The linearity of grid modulated amplifiers is not too good; this is a disadvantage in ordinary A.M. as well as in the possible use of this type of modulator for envelope restoration. It can be minimized by using suppressor or screen modulation rather than control grid modulation(11) and by careful choice and control of operating conditions.

Aside from the above limitations, there is no reason why the single sideband by envelope elimination and restoration system cannot be applied equally well to either a plate modulated transmitter or to one which is grid modulated. The phase modulated carrier may have its envelope restored in either the grid or the plate circuit — where it is done is immaterial, as long as the restored envelope is a reasonably exact copy of that originally generated.

In the latter connection, it might be well to point out that if the conventional A.M. transmitter is equipped with means for increasing the efficiency of A.M. transmission, such as frequency or volume compression or speech clipping and filtering, a definitely inexact copy of the generated S.S.B. wave will result. Unless these means were employed at the speech input to the S.S.B. generator, the compression or clipping and its distortion of the wave would be present in the A.M. branch but not in the P.M. branch. Hence it would be best to modify the A.M. transmitter to the extent that these compression or clipping circuits be bypassed or else shifted to the audio input of the S.S.B. generator.

Appendix III derives the power relationships applicable with this system when the single sideband generator is modulated by two equal amplitude audio tones of different frequency. A summary of these results follows.





Assuming a desired peak power  $P$  for a S.S.B. wave:

1. The P.M. branch must supply  $P/2.47$ , or  $0.405 P$  watts.
2. The A.M. branch must supply  $P/10.5$ , or  $0.095 P$  watts.

Taking a typical A.M. transmitter whose 100% A.M. R.F. output is 1000 watts (666 watts carrier) and whose modulator is capable of supplying at least 334 watts; application of the envelope elimination and restoration system would result in a S.S.B. wave of  $666/0.405$  or 1645 watts peak power. The A.M. modulator would have to supply only  $1645 \times 0.095$  or 157 watts.

In a normal grid modulated transmitter (A.M.) operating conditions and the load are so adjusted that at modulation peaks the instantaneous grid bias is at such a value that the R.F. component of the plate voltage equals the direct plate supply voltage. If the modulation then falls to zero (assuming linearity), the power output drops to one quarter of its maximum value and the plate efficiency falls to about 33%. Peak efficiency may go as high as 70%, but plate dissipation is high during carrier-only conditions. If such an amplifier is operating in the envelope elimination and restoration system with the output level control incorporated, it would be very nearly cut off during no-modulation conditions. Hence, under such conditions, the plate dissipation would be very nearly zero, and the power handling capabilities of the final would be increased. It seems reasonable to expect that higher power output could be achieved using the envelope elimination and restoration system with a grid modulated A.M. transmitter (preferably suppressor modulation for





better linearity) than could be expected from the same transmitter operated straight suppressor modulated A.M. This advantage of course is in addition to the multiple advantages gained in using single side-band.



## CHAPTER IV

### PRACTICAL DESIGN AND APPLICATION

The previous chapter has covered the basic principles and theory of single sideband transmission by envelope elimination and restoration, the additions to the basic circuitry necessary for practical operation with an A.M. transmitter, the power relationships, and the applicability of the system to any A.M. transmitter. In this chapter a practical system is proposed for operation with a typical A.M. transmitter; the system will be examined block by block, and design features which should be incorporated will be pointed out. Such features will be predicated on the possible use of the system with existing U. S. Navy A.M. transmitters.

The single sideband generator can be either the filter or the phasing type. In service use the filter system seems more attractive since it contains passive elements requiring little maintenance. It also will contain a minimum number of tubes and will be less critical of operating adjustment than the phasing system<sup>(4)</sup>. Since the output frequency of the S.S.B. generator will be fixed at a low radio frequency, only two frequency conversions need be made.

The following frequency selections and subsequent design features are based on the possible use of the system with the U.S. Navy type TBM, a standard 500-watt C.W., 350-watt plate modulated phone transmitter with an overall frequency range from 2 to 18.1 mc. This particular transmitter





is used as an example only to show the feasibility of adapting the envelope elimination and restoration system to a representative Navy transmitter.

The block diagram of a practical S.S.B. generator together with associated spectra is shown in Figure 9 for this system. The microphone speech level is built up by a triode class A amplifier and applied as one input to balanced modulator #1, combining it with the output of oscillator #1. The balanced modulator should be a copper oxide varistor type ring modulator, the circuit of which has been shown in Figure 2. Varistors are chosen because of their inherent stability in such circuits, for better reliability, and for space savings<sup>(4)</sup>. Oscillator #1 should have high enough output to supply about fifteen times the voltage amplitude of that supplied by the class A amplifier; this can be set by gain control of that amplifier. The bandpass filter at the chosen frequency would be a conventional L-C network rather than the more complicated crystal lattice, and can achieve 50 db or more suppression of the undesired sideband<sup>(12)</sup>. Balanced modulator #2 can be a twin triode in the conventional circuitry, fed by the output of the filter and a 400 kc crystal oscillator. The choice of oscillator frequencies allows possible frequency division and subsequent synchronization of oscillator #1 by oscillator #2. "Bandpass Filter #2" is merely a tuned circuit centered on the desired sideband and coupled to balanced modulator #2 at its output tank. It forms the grid input circuit for a pentode class A amplifier which would bring the amplitude of the signal up sufficiently for the limiter and



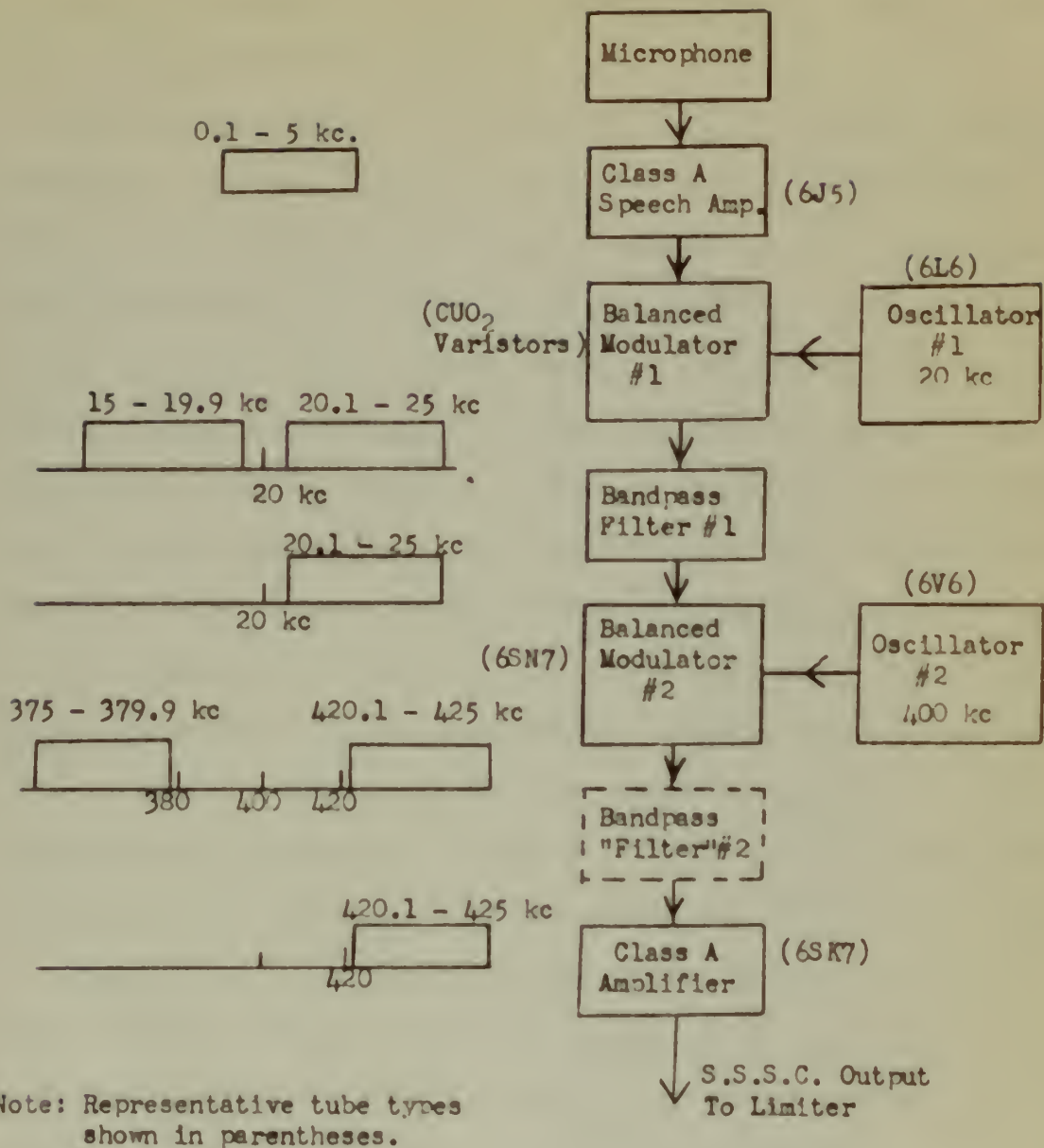


Figure 9 - Practical Single Sideband Generator





detector to use as inputs. The output of this amplifier would be S.S.B. at a carrier frequency of 420 kc. A total of five tubes would be needed for the single sideband generator.

Conservatively assuming an output of ten volts peak from the class A amplifier, the top and bottom of this wave must be heavily clipped to remove all of its amplitude variation. Clipping the wave to one tenth its peak amplitude would essentially accomplish this. A sharp cutoff pentode operated with reduced screen voltage in conventional F.M. limiter circuitry would be one approach; back-to-back diodes in the shunt type noise limiter would be another. In the F.M. circuit, the use of grid leak bias will develop additional bias voltage proportional to the excess of exciting voltage, keeping the output essentially constant.

The output of the limiter will be a pure phase modulated wave at a carrier frequency of 420 kc. Disregarding the phase equalizer portion of the chain for a moment, the operating frequency range of the entire system and the heterodyning necessary to achieve it will be considered. To do this the capabilities of the A.M. transmitter must be investigated<sup>(13)</sup>.

The first I.P.A. of the TBM transmitter has a possible frequency range of 2000 to 9050 kc. The second I.P.A. and the final amplifier may be resonated anywhere from 2000 to 18100 kc. If one additional tap were provided on plate tank L7 of the first I.P.A., all amplifiers in the transmitter could be resonated over the entire operating range. Without any modification whatever, all of the amplifiers can be tuned from 2000 to 9050 kc; hence this range will be set as the limits of the operating range of this transmitter when adapted for single sideband transmission by envelope elimination and restoration.





To obtain this range, the 420 kc phase modulated wave must be heterodyned with an oscillator running from 1580 to 8830 kc. The master oscillator of the TBM has a range from 2000 to 4525 kc. If this latter could be doubled, the total range would be the same as that of the amplifiers, that is, 2000 to 9050 kc. Using the master oscillator of the TBM as the frequency determining device and providing one external class C doubler amplifier (tunable over the 2000 to 9050 range), S.S.B. operation of the transmitter with this system would be from 2420 to 9050 kc. No higher frequency operation can be permitted since this would involve doubling in one of the transmitter class C stages, which would double the phase deviation as previously explained.

The mixer must be carefully designed to avoid intermodulation distortion, although the frequency separation between the upper and lower sum and difference frequencies is 840 kc, making it easy for the selectivity of the output circuit to keep down the unwanted beat. The mixer may be a conventional receiver type, preferably a pentagrid, separately excited mixer. The T.B.M. oscillator output of course must be greatly attenuated before application to the mixer.

The first I.P.A. of the T.B.M. transmitter is a type 860. To develop enough power to drive it, the mixer must be followed by a voltage amplifier and a power amplifier. The voltage amplifier should be a pentode for high gain, and the power amplifier (say a low drive tetrode, such as the type 6146) should be continuously tunable from 2420 to 9050 kc. Its



output should be capacity coupled through C - 15 to the grid of the first I.P.A. of the transmitter. The T.B.M. master oscillator should be disconnected at this point and its output fed to a dummy load, with a small portion of this output available for injection at the mixer.

In the A.M. chain, the detector can be completely conventional, inasmuch as it will be detecting the envelope of a single sideband wave centered in frequency in the usual receiver I.F. range. Adequate amplitude will be available at the output of the class A amplifier of the S.S.B. generator to drive a diode half wave detector. This detector output should be applied to some means of transforming its high output impedance down to a match for a 600 ohm modulator input, either with a cathode follower or a class A amplifier operating into a tube-to-line transformer. The output would be connected to the T.B.M. at terminals 1 and 2 of the modulator, the 600 ohm input to the speech amplifiers of the transmitter. The transmitter would be operated in the "manual" position, and as previously mentioned, the compression and limiting circuits should be disabled, as well as carrier control.

The phase equalizer should be designed for a total time delay exceeding the measured time delay of the modulator. It should be partially variable in small steps for fine adjustment. There are two approaches to this problem; a graded lumped circuit parameter delay line, offering the choice of many possible time delays in small steps; or a standard lumped parameter delay line, offering delays which are multiples of that given by a single section. Additional small phase shift of the







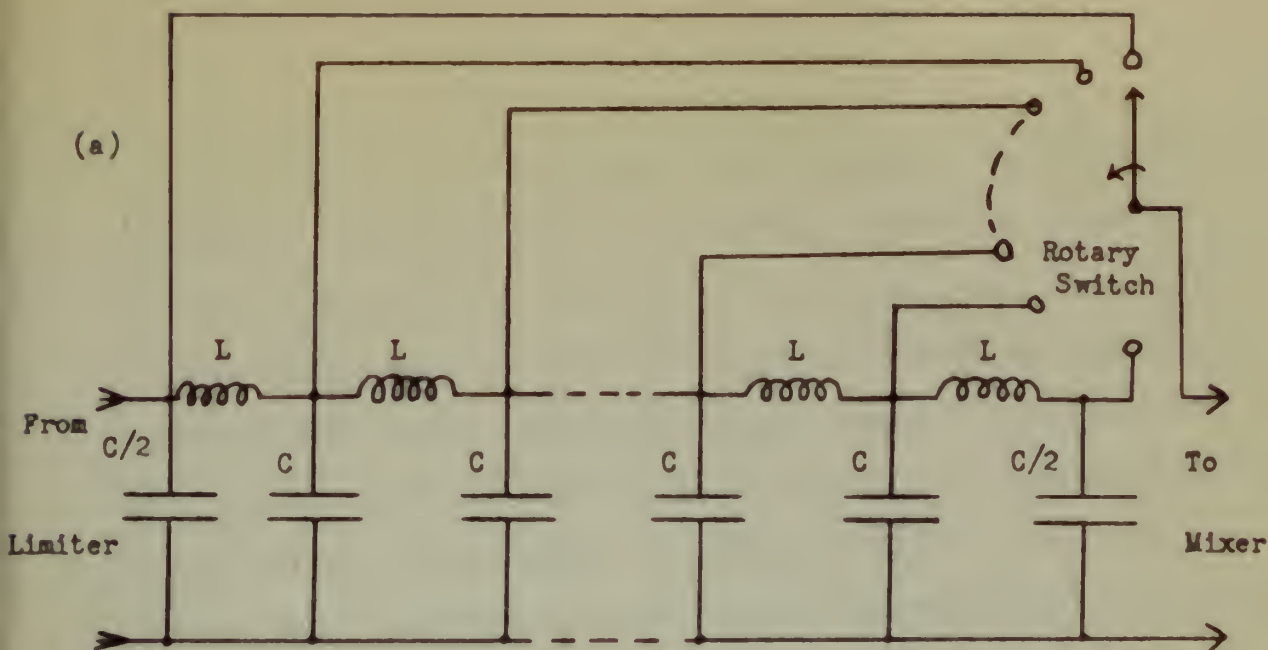
latter could be achieved with an R - C termination. Figure 10(a) and (b) show configurations for the two types.

If the time delay of the modulator is very small, compensating delay may be achieved in the P.M. branch by deliberately detuning one or more resonant circuits a slight amount. This would introduce a reactive component and subsequent time delay in the P.M. branch.

It should be noted that  $360^\circ$  of phase delay in a 420 kc P.M. branch yields only 2.4 microseconds of time delay; in the A.M. branch, assuming a modulating frequency of 1500 cps, the corresponding time delay for a  $360^\circ$  phase delay would be 667 microseconds. Therefore, if there is appreciable time delay in the A.M. branch, the delay line in the P.M. branch will have to introduce many cycles of phase delay to compensate for the modulator time delay.

If the complete system is used for frequency shift keying (achieved by alternate equal amplitude tone modulations of separate frequencies), the output will be of constant average amplitude, and the output level control will not be necessary. If voice modulation is used, the output must be of variable average amplitude to copy the S.S.B. input wave exactly, and the control must be used. It may be designed as an audio amplifier excited by the detector output, feeding a rectifier and filter to produce a negative D.C. potential whose amplitude is proportional to the average of the modulation amplitude. This negative voltage is applied to the grid of a control tube in clamp tube circuitry, capable of passing heavy

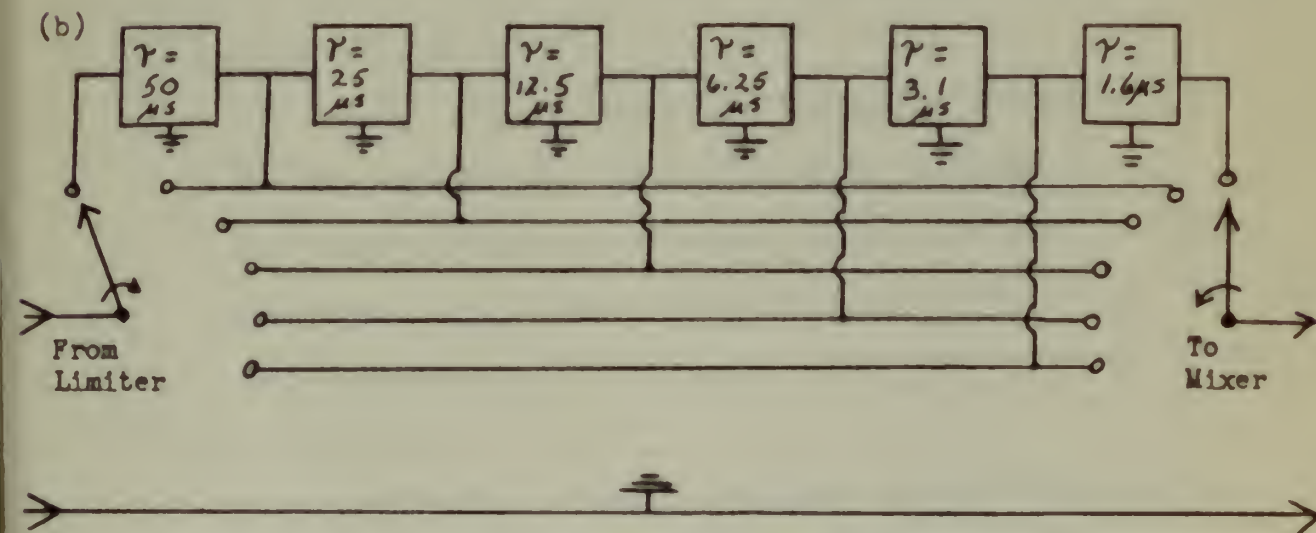




$$\gamma \text{ (delay per section)} \approx \sqrt{LC}$$

$$Z_0 \text{ of the line} \approx \sqrt{L/C}$$

Maximum delay =  $N\gamma$ , where N is number of sections used.



Same formulae as above for each section; use constant  $Z_0$ .

Maximum delay available with sample delays indicated = 98.5 microsec.

Figure 10 - Delay Line Configurations





current. The plate of this control tube is tied to terminal 28 of the transmitter, the screen grid of the 861 final amplifier. Then, as the average voice level fell, the control tube would conduct when its grid went less negative, dropping the screen voltage on the 861 and reducing its plate power input accordingly. When the negative voltage on the grid of the control tube reaches cutoff of that tube, the output of the T.B.M. will be at its maximum. Adjustment of circuit parameters must be made to set the upper and lower limits of the 861 output with desired average modulation amplitudes.

A complete block diagram showing the placement of all stages discussed is shown in Figure 11. The complete adapter less power supply has a total of 13 tubes and can be physically designed to fit into a standard receiver type cabinet. The following connections must be made to the A.M. transmitter:

- (a) Envelope input to the transmitter speech amplifier.
- (b) P.M. branch input to the grid of the first I.P.A.
- (c) Mixer injection to the master oscillator of the transmitter.
- (d) Output level control tube plate to the screen of the final P.A.

The only modifications to the A.M. transmitter necessary for use of the adapter are:

- (a) Disconnection of master oscillator output from the grid of the first I.P.A. and provision for a master oscillator dummy load.
- (b) Supply of a means for bringing out a portion of the master oscillator output to serve as the frequency determining agency for the adapter.









Use of S.S.S.C. with the T.B.M. would result in a single sideband wave output of about 1200 watts peak power; the modulator would be required to deliver about 115 watts of its total available (400 watts).

Practical adaptation of single sideband by envelope elimination and restoration could also be made to suppressor modulated transmitters. The TBL, TCK, TCN, TCU and TDE series would be Navy applications. In the plate modulated class besides the TBM are the TCA, TCB, TCC, TCF, TCP, TC8, TDF, TDH, and TDO series(14). This paper makes no attempt to outline the operational uses and advantages involved in conversion of such transmitters to single sideband emission; the conclusion drawn is that such conversion can be effected using this system should it be desirable.





## CHAPTER V

### EXPERIMENTAL RESULTS

In order to test the feasibility of adapting single sideband transmission by envelope elimination and restoration to a Navy transmitter, it was decided to try it on a low power, class B modulated equipment. The Navy type TCS-12 was chosen because of these characteristics, its availability in the laboratory, and its conventional Navy design. The transmitter consists of a master oscillator (12A6), a buffer-doubler (12A6), and a final class C amplifier (two parallel 1625s for C.W.; single 1625 for phone operation). The modulator consists of a carbon microphone transformer coupled to drive a pair of 1625s in push-pull, which affect plate and screen modulation of the single class C 1625. The transmitter is rated at 25 watts output in telegraph service and ten watts output on phone. An additional 12A6 is provided as a crystal oscillator for fixed frequency operation. The transmitter has a frequency range of 1500 kc to 12,000 kc<sup>(15)</sup>.

#### A. Design Features

The single sideband generator that was used was of the crystal filter type. Output of the generator was at 450 kc. It employed a 6AU6 speech amplifier, one half of a 12AU7 as a carrier oscillator, a 12AU7 balanced modulator, and a single channel crystal filter rejecting the upper sideband. With two tone modulation applied, it produced a single sideband output of about six volts peak.



Figure 12 shows the complete experimental circuit used for the adapter. The limiter required the given values of screen, plate load, and grid leak resistances to start limiting at about 1 volt R.M.S. of input signal. Resulting screen voltage was 40 volts, and plate supply voltage was 150 volts, with  $E_{bb} = 250$  volts. Essentially all of the amplitude variation in a two tone input S.S.B. wave was removed by the limiter, as shown by oscilloscope observation. The output stabilized after limiting at about four volts peak.

The mixer was conventional as shown; a separate triode in Pierce circuitry was employed as a crystal oscillator for injection for test convenience, although the TCS master oscillator would have worked equally well. The phase equalizer was omitted in order to check on its necessity.

The voltage amplifier using the 6AC7 gave the usual self-oscillation troubles; it was finally stabilized by using a 6SJ7, giving less gain, but adequate for the purpose. Resonant circuits at the output frequency of 3334.5 kc consisted of 100 mmf midget air variable capacitors across one inch diameter close wound coils (35 turns of #26 enamel wire) on phenolic plug-in forms. The output of the voltage amplifier was taken from a link on the final coil.

The detector was a half-wave diode as illustrated, and its input was paralleled with that of the limiter. Naturally, this resulted in detuning both inputs, but adequate excitation was obtained with this experimental setup, so it was not changed.







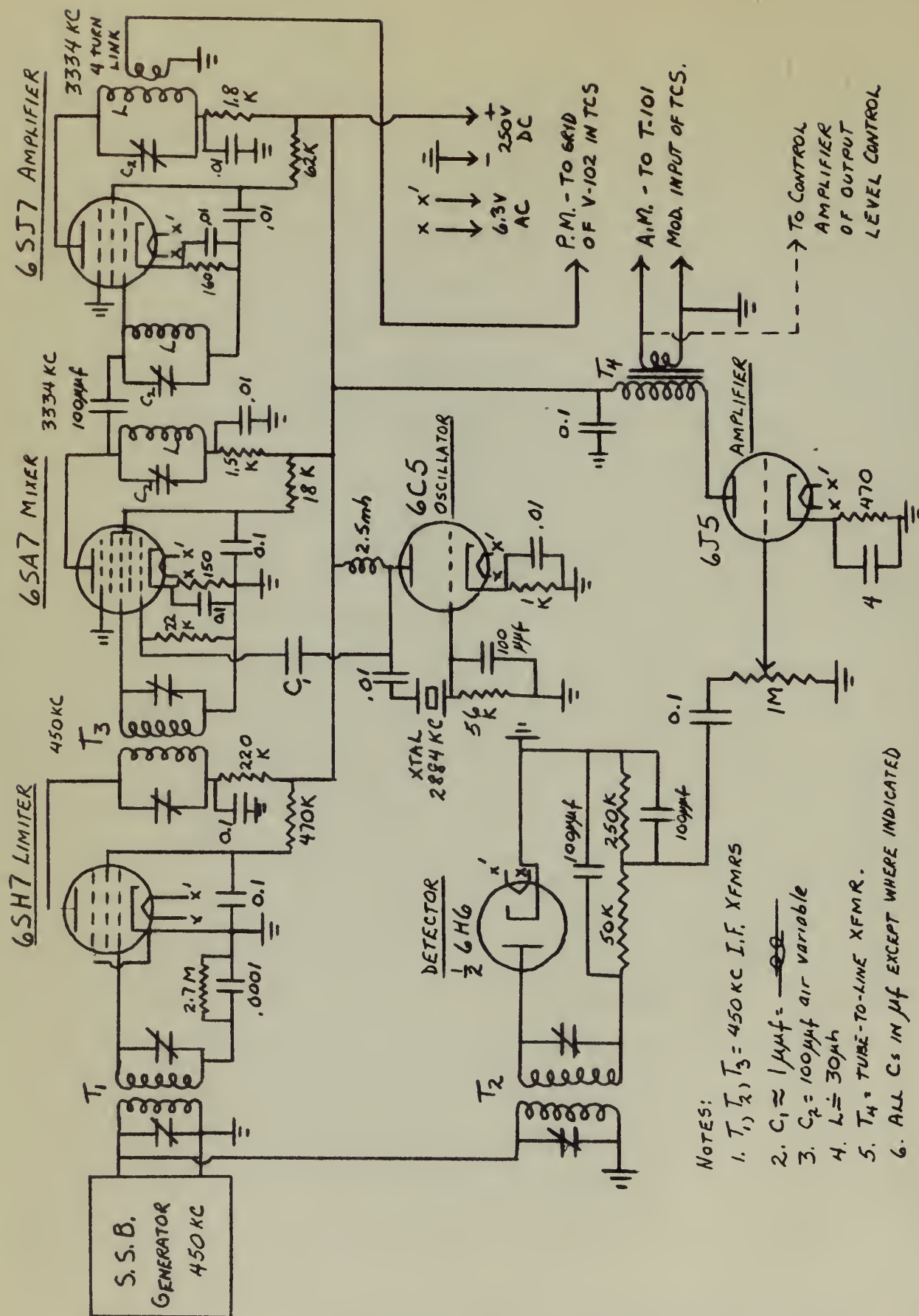


FIGURE 12 - EXPERIMENTAL ADAPTER; COMPLETE CIRCUIT

A cathode follower was first built up to follow the detector and effect the desired impedance transformation; this proved to supply insufficient audio output and was changed to a triode class A amplifier with a tube-to-line transformer plate load.

Preliminary tests of the completed adapter indicated (with a two-tone S.S.B. test input) pure phase modulation of about 20 volts peak at the output of the voltage amplifier and a good reproduction of the input envelope at the detector output.

The completed system was then connected to the TCS-12 transmitter as follows: the link output of the 6SJ7 (P.M.) was plugged into crystal socket #1 of the transmitter, thus exciting the grid of V-102, the crystal oscillator. The audio output from the secondary of the tube-to-line transformer was connected between pin #2 of T-101 (the TCS-12 microphone transformer) and ground. Adequate excitation was obtained from the 6SJ7 voltage amplifier to drive the TCS-12 to full output in the C.W. position. The output of the transmitter was dissipated into a standard Ohmite 13 ohm dummy load. The complete experimental setup, including the modulating agencies used, is shown in the block diagram of Figure 13. The transmitter was tuned for straight-through operation at the output frequency of the 6SJ7 voltage amplifier, 3334.5 kc.

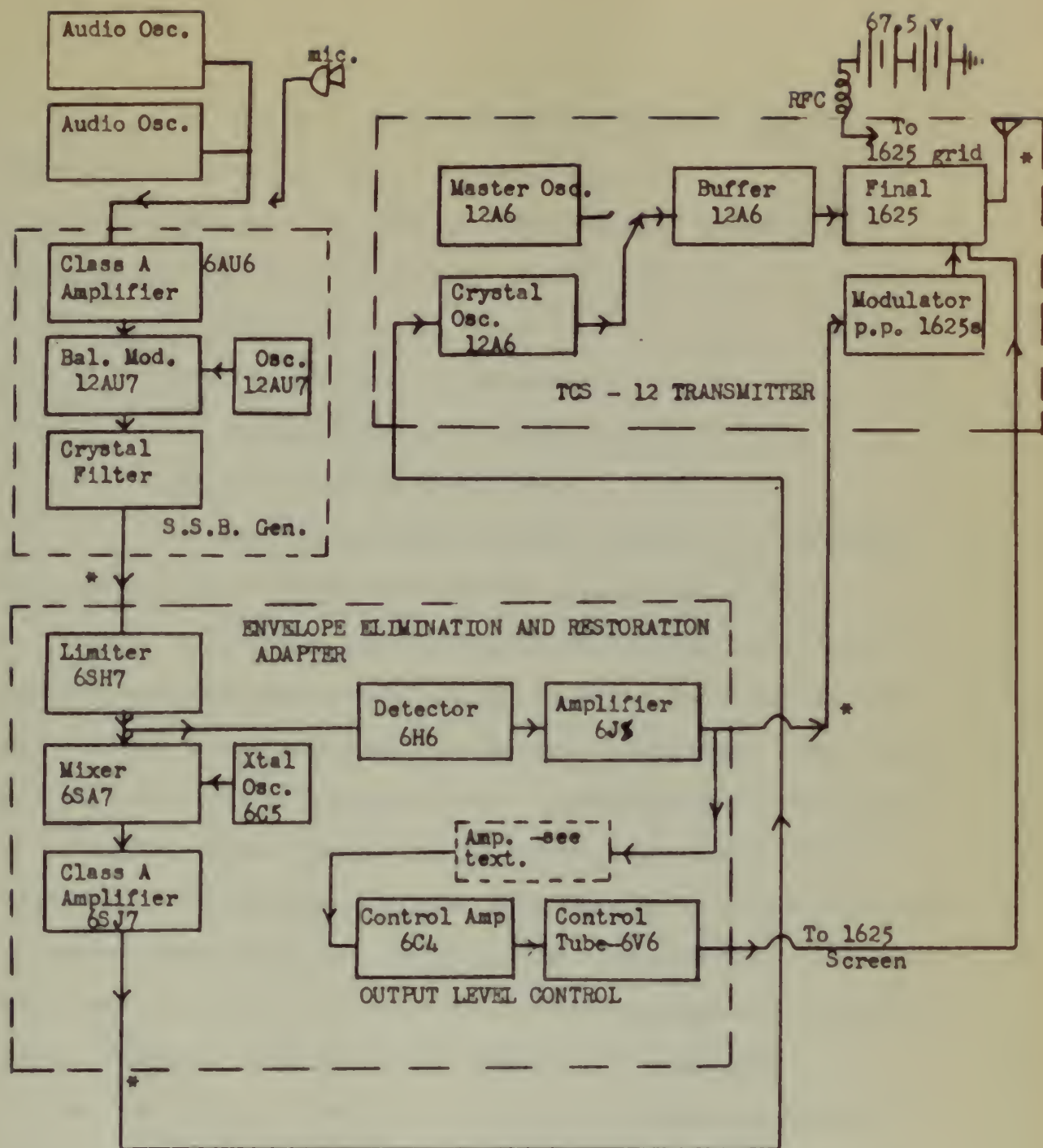
#### B. Test Results

1. With single tone modulation of the S.S.B. generator, transmitter fully loaded, emission switch in "Voice" position.

- a. Effective measured power output = 12 watts.







\* Waveforms taken at these points with a Tektronix Type 513D Oscilloscope.

Figure 13 - Experimental Circuits Used; Block Diagram





b. Emission: Single sideband; a pure R.F. emission below the carrier frequency by a frequency  $f_m$ .

2. With two tone (1500 and 2000 cps) equal amplitude (0.2 volts R.M.S.) audio input to the S.S.B. generator:

- a. Peak power - 43.5 watts
- b. Effective power - 21.8 watts
- c. Average power - 17.5 watts
- d. Modulator power - 4.1 watts
- e. Emission - Single sideband, as proven by resupplying the carrier at the receiver and obtaining the two tone output.

Carrier and sideband suppression were evident when monitoring the emission with a standard communications receiver but were not measured, since the aim of this experiment was merely to prove that a single sideband wave was produced using this system in conjunction with the TCS-12.

The power figures obtained in test #2 are based on peak-to-peak measurements of waveforms taken with a Tektronix oscilloscope and on the methods of calculations used in Appendix III. The power factor of the dummy load was taken into account; the load was measured on a bridge to have an impedance at the operating frequency of 12 plus j20.

The time delay at a modulating frequency of 500 cycles between the input and the output of the modulator was measured by oscilloscope comparison of waveforms; it proved to be too small to be measurable by this method. This accounts for the recognizable S.S.B. output obtained without the use of the phase equalizer. Since the TCS-12 modulator contains only one stage, this insignificant time delay is understandable.

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The next phase of the experiment consisted of modifying the system for transmission of variable average amplitude voice transmission. An output level control was constructed, the circuit of which appears in Figure 14. The audio (obtained from the amplified output of the detector) is further amplified by the 6C4 and is transformer coupled to a diode rectifier (1N34). Instantaneous audio variations at the output of this diode are filtered out by the capacitor and resistor, leaving a D. C. negative voltage at the grid of the 6V6 which is proportional to the average amplitude of the original audio impressed. The 6V6 control tube plate is connected to the screen grid of V-104 in the TCS-12 and thus acts as a clamp on that potential. When the average voice amplitude is high, a large negative voltage keeps the control tube cut off, and the screen of the final amplifier in the transmitter is at normal operating potential, permitting full output of that stage. The magnitude of the audio level bringing about this condition is set by variation of the 500 K input potentiometer. When the average audio level falls, the negative D.C. voltage on the grid of the control tube falls also, allowing it to conduct. This conduction drops the screen voltage on the final amplifier in the transmitter, since increased current through the screen dropping resistor in the TCS-12 will cause a greater voltage drop across that resistor. Therefore, the average power output of the transmitter will vary with the average audio level. The minimum power output of the transmitter with no modulation can be set (within limits) by variation of the cathode potentiometer of the control tube.





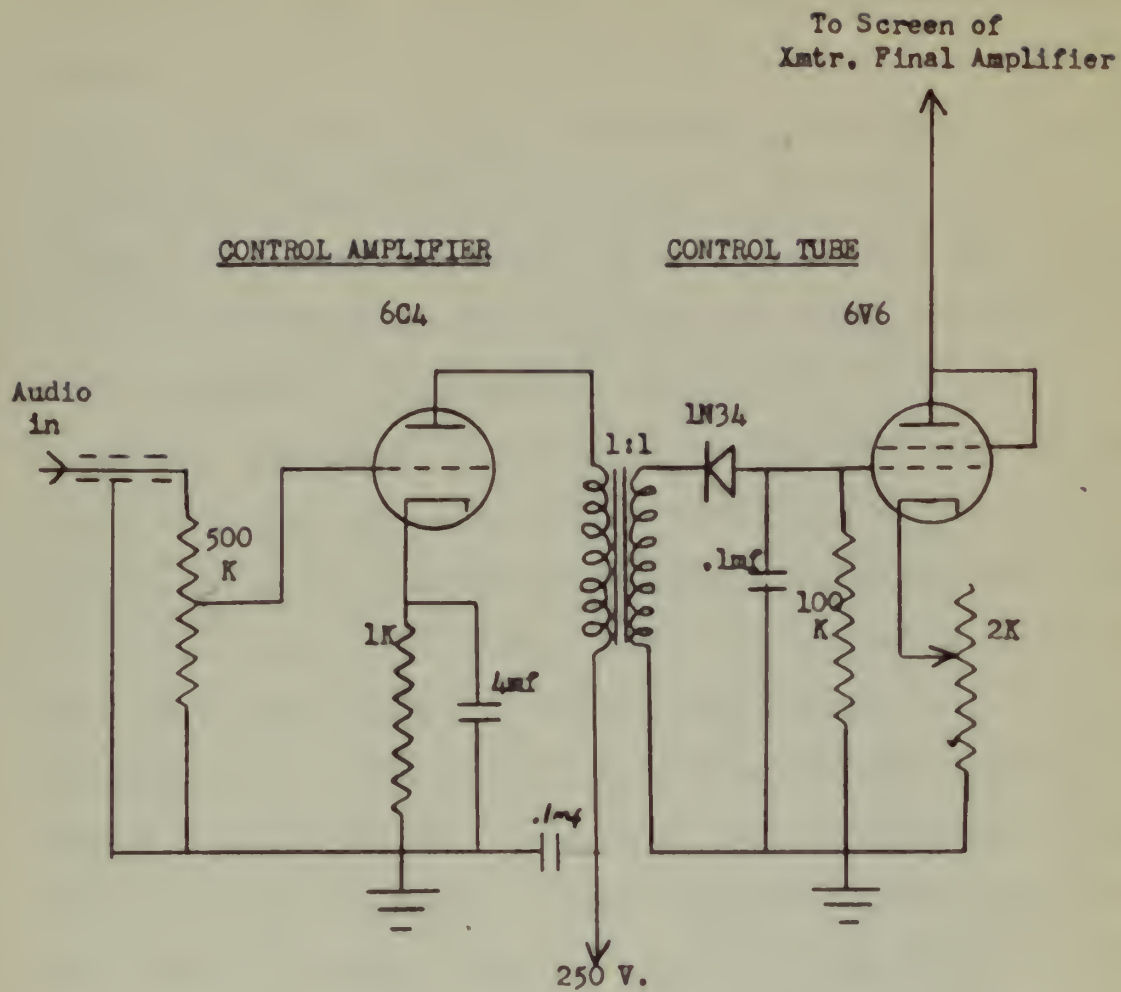


Figure 14 - Output Level Control





The output level control required two modifications to the TCS-12. The plate of the control tube was connected to the screen side of resistors R-108 and R-109; and fixed bias on the final amplifier had to be provided, since loss of operating bias during periods of no modulation would have nullified the effect of the output level control. The TCS-12 is provided with operating bias only; hence a battery bias of - 67.5 volts was connected through an R.F. choke to the junction of R-123 and R-107 in the grid circuit of the final amplifier. This bias effectively cut off the final when all excitation was lost.

On the first test of the system, it was found that there was insufficient audio excitation at the grid of the control amplifier to cut off the control tube with full modulation applied. Hence additional amplification was placed in the circuit in the form of a Ballantine meter amplifier, indicated in Figure 13. When this was adjusted for an absolute voltage gain of 5, sufficient D.C. was available with full modulation applied to cut off the 6V6. The complete system was then put in operation and the S.S.B. generator was voice modulated. TCS-12 output waveforms on the oscilloscope indicated a varying average amplitude output, and transmitter metering also confirmed this. The signal was tuned in on a communication receiver, and using normal single sideband reception techniques (resupplying the carrier with the receiver beat frequency oscillator), a completely intelligible speech output was obtained. Further verification that the received emission was true S.S.S.C. was obtained through use of a YRS-1 receiver adapter. This adapter used in conjunction



with a receiver locks in on the suppressed carrier and provides separate channels for the upper and lower sidebands. When switched for reception of the upper sideband after lock-in, no output was obtained. Switching to the lower sideband provided completely intelligible speech output. Thus the aim of the experiment was achieved; to produce an amplified copy of a single sideband suppressed carrier wave using the adapter and a standard U. S. Navy transmitter.

### C. Recommendations

Naturally, with the purely experimental circuitry described, several practical difficulties were encountered in the course of the experiment. The solution of these difficulties lies in the application of practical engineering techniques, which were minimized during this experiment only in the interests of expediency. The entire adapter should be shielded, with interstage shielding around all radiating stages and especially around the 6SJ7 voltage amplifier. Shielded cabling should be used for all connections. The audio branch of the adapter was underpowered; pentode class A amplifiers should be used instead of triodes. A better scheme for coupling the S.S.B. generator to the detector and the limiter should be devised; an I.F. transformer with a single primary and twin secondaries might be designed. The entire adapter, including the single sideband generator and the output level control, should be built up on one chassis; substitution of all miniature tubes and judicious grouping of stages would easily permit construction on a chassis 13" x 17" x 3" or smaller. Many of the components, such as transformers,







were chosen on the basis of immediate availability and were not necessarily the correct design values. After complete engineering of the adapter, tests for intermodulation distortion, carrier suppression, and sideband suppression should be made.

From the experiment, then, it may be concluded that single sideband transmission by envelope elimination and restoration using a U. S. Navy standard A.M. transmitter is completely realizable. This system appears to be worthy of serious consideration in any case where a relatively high power single sideband emission is desired, with only A.M. equipment on hand. With the ever-increasing use of single sideband for high-efficiency transmission of intelligence on point-to-point circuits, it seems logical that this system will have application.



POOR BIBLIOGRAPHIC ARRANGEMENT.

1. FORM IS NOT PROPER
2. ARRANGEMENT SHOULD BE AN  
ALPHABETICAL ONE.

(SEE GLENDINNING'S THESIS)



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Transmitter-Receiver



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## APPENDIX I

### AMPLITUDE AND PHASE MODULATION

#### I. Amplitude Modulation.

A. Definition: An amplitude modulated wave is a radio frequency emission in which the intelligence is conveyed by variation of the amplitude of the radio frequency wave.

#### B. Derivation:

1. Given: R.F. carrier wave  $\hat{E}_c \sin \omega_c t$

Audio intelligence  $\hat{E}_m \cos \omega_m t$  (single tone)

2. Let the carrier amplitude vary at the audio rate, giving

$$e = [\hat{E}_c + k \hat{E}_m \cos \omega_m t] \sin \omega_c t$$

$$(a) \quad e = \hat{E}_c \left[ 1 + \frac{k \hat{E}_m}{\hat{E}_c} \cos \omega_m t \right] \sin \omega_c t$$

3. This may be expanded to give

$$(b) \quad e = \hat{E}_c \sin \omega_c t + \frac{m_a \hat{E}_c}{2} \sin 2\pi(f_c + f_m)t + \frac{m_a \hat{E}_c}{2} \sin 2\pi(f_c - f_m)t$$

where  $100m_a = 100k \frac{\hat{E}_m}{\hat{E}_c}$ , the percent modulation. The first term is the R.F. carrier, the second term the upper sideband, and the third term the lower sideband.

4. Let  $m_a = 1$  (100% modulation); then (b) becomes

$$(c) \quad e = \hat{E}_c \sin \omega_c t + \frac{\hat{E}_c}{2} \sin 2\pi(f_c + f_m)t + \frac{\hat{E}_c}{2} \sin 2\pi(f_c - f_m)t$$

#### C. Power Considerations.

1. Assuming (c) is dissipated in a resistance R, then

$$(a) \text{ Carrier power} = \frac{\hat{E}_c^2}{2R}$$

$$(b) \text{ Upper sideband power} = \frac{\hat{E}_c^2}{8R}$$

$$(c) \text{ Lower sideband power} = \frac{\hat{E}_c^2}{8R}$$

$$(d) \text{ Total power} = 1\frac{1}{2} \times \frac{\hat{E}_c^2}{2R}$$



## II. Phase Modulation.

A. Definition: Phase modulation is that type of modulation produced by varying the instantaneous phase angle of the R.F. wave, the instantaneous deviation of the phase angle from the unmodulated value being directly proportional to the instantaneous value of the modulating wave but independent of its frequency. (10)

B. Derivation:

1. Given: R.F. carrier wave  $\hat{E}_c \sin \omega_c t$   
 Audio intelligence  $\hat{E}_m \cos \omega_m t$  (single tone)

2. Let the phase angle of the carrier vary at the audio rate (assumed zero in the unmodulated state for simplicity). Then the equation for a phase modulated wave is

$$(a) \quad e = \hat{E}_c \sin [\omega_c t + k \hat{E}_m \cos \omega_m t]$$

3. Expanding this, the following components are derived: (6)

$$\begin{aligned} e &= \hat{E}_c \left\{ \sin \omega_c t \cos [k \hat{E}_m \cos \omega_m t] + \cos \omega_c t \sin [k \hat{E}_m \cos \omega_m t] \right\} \\ &= \hat{E}_c \left\{ \sin \omega_c t [J_0(k \hat{E}_m) - 2 J_2(k \hat{E}_m) \cos 2 \omega_m t + 2 J_4(k \hat{E}_m) \cos 4 \omega_m t - \dots] \right. \\ &\quad \left. + \cos \omega_c t [2 J_1(k \hat{E}_m) \cos \omega_m t - 2 J_3(k \hat{E}_m) \cos 3 \omega_m t + \dots] \right\} \\ \text{Carrier:} \quad &= \hat{E}_c \{ J_0(k \hat{E}_m) \sin \omega_c t \} \end{aligned}$$

$$\begin{aligned} \text{1st Order Sidebands:} \quad &+ \hat{E}_c \{ J_1(k \hat{E}_m) \cos [\omega_c + \omega_m] t + J_1(k \hat{E}_m) \cos [\omega_c - \omega_m] t \} \\ \text{2nd Order Sidebands:} \quad &- \hat{E}_c \{ J_2(k \hat{E}_m) \sin [\omega_c + 2 \omega_m] t + J_2(k \hat{E}_m) \sin [\omega_c - 2 \omega_m] t \} \\ \text{3rd Order Sidebands:} \quad &- \hat{E}_c \{ J_3(k \hat{E}_m) \cos [\omega_c + 3 \omega_m] t + J_3(k \hat{E}_m) \cos [\omega_c - 3 \omega_m] t \} \\ \text{4th Order Sidebands:} \quad &+ \hat{E}_c \{ J_4(k \hat{E}_m) \sin [\omega_c + 4 \omega_m] t + J_4(k \hat{E}_m) \sin [\omega_c - 4 \omega_m] t \} \\ &\dots \dots \dots \text{etc.} \end{aligned}$$





4. Note in the foregoing that the first order P.M. sidebands produced by a modulating signal  $\hat{E}_m \cos \omega_m t$  are in R.F. phase quadrature with the sidebands produced in A.M. by the same modulating signal (shown in B (3)). Also note that each successively higher order P.M. sideband lags its predecessor by  $90^\circ$ .

C. Instantaneous frequency of P.M. wave.

1. Let  $\phi = \omega_c t + K \hat{E}_m \cos \omega_m t$

2. Then the instantaneous frequency of the wave  $e^{j\phi}$  is  $\frac{1}{2\pi} \frac{d\phi}{dt}$

or  $f_{inst.} = \frac{1}{2\pi} \frac{d}{dt} \{ \omega_c t + K \hat{E}_m \cos \omega_m t \}$

3. Differentiating as indicated,

$$\begin{aligned} f_{inst.} &= \frac{1}{2\pi} \{ 2\pi f_c - K \hat{E}_m 2\pi f_m \sin \omega_m t \} \\ &= f_c - K \hat{E}_m f_m \sin \omega_m t \end{aligned}$$



## APPENDIX II

### DERIVATION OF INSTANTANEOUS FREQUENCY OF A SINGLE SIDEBAND WAVE WITH CARRIER MODULATED BY A SINGLE FREQUENCY AUDIO TONE

Given the S.S.B. wave with carrier (derived in Chapter III)

$$\sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \omega_m t} \cos \left[ \omega_c t + \arctan \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t} \right]$$

To find its instantaneous frequency  $f_i$ :

$$\omega_i = \frac{d}{dt} \left[ \omega_c t + \arctan \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t} \right]$$

$$f_i = \frac{1}{2\pi} \frac{d}{dt} \left[ \omega_c t + \arctan \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t} \right]$$

$$= f_c + \frac{\frac{1}{2\pi} \frac{d}{dt} \left[ \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t} \right]}{1 + \left[ \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t} \right]^2}$$

$$\text{where: } \frac{d}{dt} \left[ \frac{A_2 \sin \omega_m t}{A_1 + A_2 \cos \omega_m t} \right] = \frac{(A_1 + A_2 \cos \omega_m t) A_2 \omega_m \cos \omega_m t - A_2 \sin \omega_m t (-A_2 \omega_m \sin \omega_m t)}{(A_1 + A_2 \cos \omega_m t)^2}$$

$$= 2\pi f_m \left\{ \frac{A_1 A_2 \cos \omega_m t + A_2^2 \cos^2 \omega_m t + A_2^2 \sin^2 \omega_m t}{(A_1 + A_2 \cos \omega_m t)^2} \right\}$$

$$= 2\pi f_m \left\{ \frac{A_2^2 + A_1 A_2 \cos \omega_m t}{(A_1 + A_2 \cos \omega_m t)^2} \right\}$$

substituting:

$$f_i = f_c + f_m \left\{ \frac{A_2^2 + A_1 A_2 \cos \omega_m t}{(A_1 + A_2 \cos \omega_m t)^2} \right\} \times \left\{ \frac{(A_1 + A_2 \cos \omega_m t)^2}{(A_1 + A_2 \cos \omega_m t)^2 + A_2^2 \sin^2 \omega_m t} \right\}$$

$$= f_c + A_2 f_m \left\{ \frac{A_2 + A_1 \cos \omega_m t}{A_1^2 + A_2^2 + 2A_1 A_2 \cos \omega_m t} \right\}, \text{ the instantaneous frequency of the S.S.B. wave}$$



APPENDIX III  
SINGLE SIDEBAND SUPPRESSED CARRIER WAVE  
PRODUCED BY TWO EQUAL AMPLITUDE AUDIO  
TONES - POWER RELATIONSHIPS

With complete carrier suppression assumed as an approximation, the wave produced by a single sideband generator modulated by two equal amplitude audio tones of different frequency may be expressed as

$$e = E \cos \omega_1 t + E \cos (\omega_1 + \omega_2) t$$

As shown in Chapter III, the amplitude of this wave is

$$\begin{aligned} \sqrt{E^2 + E^2 + 2E^2 \cos \omega_2 t} &= E \sqrt{2(1 + \cos \omega_2 t)} \\ &= 2E \sqrt{\frac{1 + \cos \omega_2 t}{2}} = 2E \cos \frac{\omega_2 t}{2} \end{aligned}$$

The peak amplitude therefore is  $2E$ .

Assuming power contained in the wave is dissipated in a resistance  $R$ , then the peak power =  $4E^2/R = \hat{P}$ .

The average power dissipated (since this is a cosinusoidal wave) is

$$\left[ \frac{2}{\pi} \right]^2 \frac{4E^2}{R} = \frac{1.62E^2}{R} = P_{av}$$

This will be the power in the phase modulated component of the S.S.B. wave, or the power supplied by the transmitter's P.M. branch.

$$\text{Then } \frac{\hat{P}}{P_{av}} = \left[ \frac{\pi}{2} \right]^2 = 2.47$$

(Compared to 4 for A.M. plate modulation)

The total or effective power dissipated in  $R$  by the wave is

$$(0.707)^2 \hat{P} = \frac{1}{2} \hat{P} = \frac{2E^2}{R} = P_{eff}$$

Then

$$P_{eff} - P_{av} = \frac{2E^2}{R} - \frac{1.62E^2}{R} = \frac{0.38E^2}{R} = P_{mod}$$

which will be the remainder, or the power that must be supplied by the modulator. Therefore

$$\frac{\hat{P}}{P_{mod}} = \frac{4}{0.38} = 10.5$$





From the foregoing it is seen that

Power supplied by the P.M. branch is  $\hat{P}/2.47$

Power supplied by the A.M. branch is  $\hat{P}/10.5$

Example (using the two tone test signal):

If a one kilowatt peak power S.S.B. signal is desired, the R.F. or P.M. branch of the transmitter supplied  $1000/2.47$  or 405 watts. The modulator, or A.M. branch supplies  $1000/10.5$  or 95 watts.













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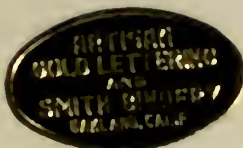
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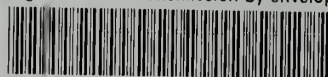
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